**MathLinks****6-11**

STUDENT PACKET

**MATHLINKS: GRADE 6  
STUDENT PACKET 11  
RATIOS AND UNIT RATES**

<b>11.1</b>	<b>Ratios</b> <ul style="list-style-type: none"><li>• Define ratio terminology.</li><li>• Explore equivalent ratios.</li><li>• Represent ratios using symbols, words, tables, and tape diagrams.</li><li>• Solve problems using tables and tape diagrams.</li></ul>	<b>1</b>
<b>11.2</b>	<b>Unit Rates</b> <ul style="list-style-type: none"><li>• Relate unit rate to ratio.</li><li>• Represent rates using symbols, words, tables, and double number line diagrams.</li><li>• Solve problems using rates, tables and double number line diagrams.</li></ul>	<b>9</b>
<b>11.3</b>	<b>Ratio and Unit Rate Problems</b> <ul style="list-style-type: none"><li>• Solve ratio and unit rate problems using a variety of strategies.</li></ul>	<b>17</b>
<b>11.4</b>	<b>Skill Builders, Vocabulary, and Review</b>	<b>25</b>

Commentary on the packet will be in red in text boxes along the way.

Welcome to a *MathLinks* Student Packet (SP). This packet is from *MathLinks: Grade 6* and is SP11, meaning it is the 11<sup>th</sup> packet out of 16.

On the cover sheet you will find the titles, goals, and page numbers of the three concept lessons as well the location of the fourth lesson which is always the Skill Builder, Vocabulary, and Review.

## WORD BANK

Word or Phrase	Definition or Description	Example or Picture
double number line diagram		<div style="border: 1px solid black; padding: 10px; color: red;"> <p>All major vocabulary for the SP is found in the Word Bank, though some words are introduced and defined within the lessons. All words are defined or explained in Resource Guide.</p> <p>The Resource Guide also includes explanations and examples. It replaces the examples and glossary of a traditional textbook.</p> <p>Students will receive the resource guide in two parts, roughly corresponding to the two semesters in the school year.</p> </div>
equivalent ratios		
ratio		
tape diagram		
unit price		
unit rate		
value of a ratio		

# RATIOS

## Summary

We will define ratio and explore when ratios are equivalent. We will represent ratios using tables and diagrams, and solve problems involving ratios.

## Goals

- Define ratio terminology.
- Explore equivalent ratios.
- Represent ratios using symbols, words, tables, and tape diagrams.
- Solve problems using tables and tape diagrams.

## Warmup

1. Gretchen was asked to write three fractions that are equivalent to  $\frac{3}{7}$ . Her work is shown below.

$$\frac{3}{7} + \frac{2}{2} = \frac{5}{9}$$

$$\frac{3}{7} + \frac{3}{3} = \frac{6}{10}$$

$$\frac{3}{7} + \frac{4}{4} = \frac{7}{11}$$

Explain Gretchen's mistaken thought process.

The black strip along the top of this page, along with the Summary and Goals of this lesson, signifies the beginning of a new lesson.

All lessons begin with a Warmup that reviews or previews knowledge for the new lesson.

2. An old television commercial stated that 4 out of 5 dentists sur  
sugarless gum for their patients who chew gum.

Explain, in your own words, what you think this statement means.

## INTRODUCTION TO RATIOS

A ratio is a pair of numbers, not both zero, in a specific order.

The ratio of  $a$  to  $b$  can be denoted by  $a : b$  (read “ $a$  to  $b$ ,” or

Example: If there were 3 coins and 2 paperclips in your pocket, the ratio of the number (#) of coins to the number (#) of paperclips is 3 to 2 or this ratio simply as “the ratio of coins to paperclips.”

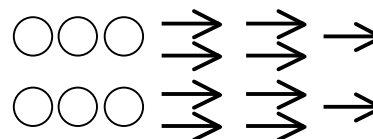
Though none are intended to be, some pages seem more like a workbook than others. We establish some definitions and apply them here.

In the Teacher Packet (TP), which is in the Teacher Guide, you will find information regarding this lesson.

Write the ratios below for this diagram of circles and arrows.

1.	Number of circles to number of arrows	3 to ___ or 3 : ___
2.	Number of circles to total number of shapes	3 to ___ or ___ : ___
3.	Number of arrows to number of circles	___ to ___ or ___ : ___
4.	Number of arrows to total number of shapes	___ to ___ or ___ : ___
5.	Total number of shapes to number of arrows	___ to ___ or ___ : ___
6.	Number of circles to number of circles	___ to ___ or ___ : ___

7. The original picture is repeated twice here. There are still 3 circles for every 5 arrows.

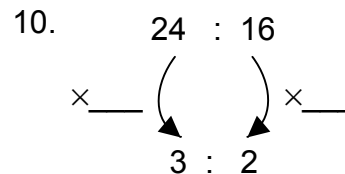
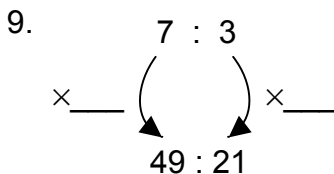
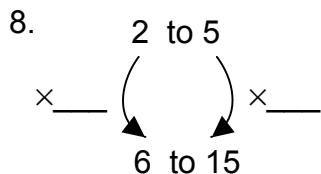


a. The new circle to arrow ratio is 6 : \_\_\_

b. Each number in the ratio 3 : 5 can be multiplied by what number to obtain this new ratio? \_\_\_\_\_

Two ratios are equivalent if each number in one ratio is a multiple of the corresponding number in the other ratio by the same positive number.

In each arrow diagram below, write the multiplier that can be used to justify that the ratios are equivalent.



## EQUIVALENT RATIOS IN TABLES

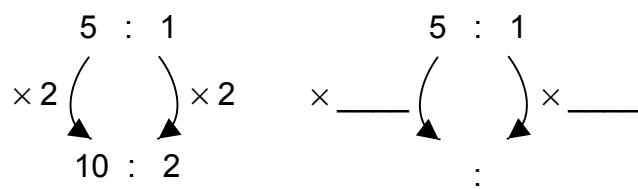
When variables in a fixed ratio are represented in tables, pairs of table entries form equivalent ratios.

1. The teacher said that the ratio of the number of fish to the number of frogs in the science lab is 5 to 1, or 5 fish for every 1 frog.

a. Complete a horizontal table below for possible numbers of animals that could be in the lab.

<b># of fish</b>	5	10	30		
<b># of frogs</b>	1			20	
<b>Total</b>					600

b. The ratios 5 : 1 and 10 : 2 are equivalent ratios because each number in the first pair is multiplied by 2 in the second pair. Using the columns in the table above and the arrow diagrams below, write another ratio that is equivalent to 5:1.



c. Using columns in the table above, write two ratios that represent the ratio of the number of fish to the total number of animals. Show the multiplier with an arrow diagram.

\_\_\_\_\_ to \_\_\_\_\_

\_\_\_\_\_ to \_\_\_\_\_

2. The ratio of the number of 12-year-olds to the number of 11-year-olds in the soccer tournament is 1 to 2.

a. Create a vertical table to the right for this situation.

b. Using rows in the table, write two different ratios that are equivalent to 1 : 2. Show they are equivalent with arrow diagrams.

# of 11-year olds	# of 12-year olds	Total

c. The ratio of the number of 11-year olds to the total number of tournament players is \_\_\_\_\_ to \_\_\_\_\_. Choose two ratios for this situation from your table and explain with diagrams and words why they are equivalent.

## EXPLORING RATIOS

- For an art project in all of her classes, a teacher wants each group to have 4 rulers and 3 glue sticks. What is the ratio of rulers to glue sticks needed for this project?
- Rodrigo passes out supplies in Period 1. He gives 4 rulers and 3 glue sticks to the first group, 4 rulers and 3 glue sticks to the next group, and continues until he has distributed 24 rulers and a certain number of glue sticks.
  - How many glue sticks did he distribute? \_\_\_\_\_
  - Use an arrow diagram to show that a ratio formed by the number of rulers and glue sticks distributed is equivalent to the ratio in Problem 1.
- Pauline passes out supplies to seven groups in Period 2. She says, "I will quickly calculate the numbers of rulers and glue sticks distributed to all the groups first, and then pass them out." Show using ratios how Pauline determined the number of rulers and glue sticks needed.

This page is intended to be done with partners so that discussion is included.

- Examine the rows in the table to the right. Are the ratios of feet to eyes equivalent for fish, cows and chicks? Explain.

	# of feet	# of eyes
<b>fish</b>	0	2
<b>cows</b>	20	10
<b>chicks</b>	48	48

- Examine the rows in the table to the right. Are the ratios of fingers to ears equivalent in each period? Explain.

	# of fingers	# of ears
<b>period 1</b>	50	10
<b>period 2</b>	170	34
<b>period 3</b>	250	50
<b>period 4</b>	150	30



## GRAPE JUICE MIXTURES

Your teacher will give you cards that indicate ways to combine cups of grape concentrate and cups of water to form grape juice mixtures. Discuss the questions below with your partners and record your thinking.

1. Arrange the cards from least “grapey” to most “grapey” and record the letters on the cards here. Note that some of the mixtures may be “equally grapey,” or equivalent.

\_\_\_\_\_ least \_\_\_\_\_ most \_\_\_\_\_

2. Choose a pair of cards and explain how you know which

Critique the reasoning of each student.

3. Jody said, “Mixture A and mixture J will taste the same because they have the same number of cups of grape.”

4. Ed said, “Mixture E and mixture C will taste the same because they have the same amount of water than grape, and mixture C has one more cup of

This page is to be completed using the Grape Mixture Reproducible. Students work and discuss with a partner how to compare various mixtures, then the put them in order from least to most grapey. The reproducible and discussion drive this lesson.

BTW – We attempted to keep reproducibles to a minimum so that you don’t have lots to copy. Reproducibles are located in the back of the Teacher Guide.

5. Cary said, “Mixture C is more grapey than mixture J because it has more cups of grape.”

6. Jon said, “For any mixture, if I double the number of cups of grape and double the number of cups of water, the taste will be the same.”

Determine which the following are more, less, or equally grapey when compared to mixture J (2 cups grape : 1 cup water). Explain your answers.

7. The mixture is changed to 2 **ounces** grape : 1 **ounce** water.
8. The mixture is changed to 2 **gallons** grape : 1 **gallon** water.
9. The mixture is changed to 2 **ounces** grape : 1 **gallon** water.



# TAPE DIAGRAMS

A tape diagram is a graphical representation of two variables in which rectangles of equal area represent equal quantities. Tape diagrams are typically used when the quantities have the same units.

The pictures below are both appropriate tape diagrams to represent Card A.

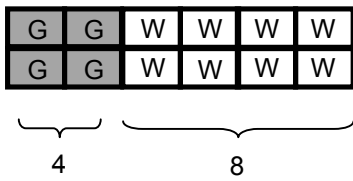
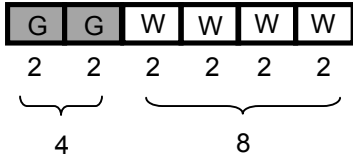


Tape diagrams are introduced here. They occur naturally because units are the same.

Draw tape diagrams to represent each indicated juice mixture.

1. Card J	2. Card B
3. Card F	4. Card H

Alex and Andrea were asked how many gallons of grape concentrate and water were needed to make 12 gallons of juice that tastes like mixture A. Their diagrams are below.

<p>5. Explain Alex's method:</p> 	<p>6. Use Alex's method to find the amount of each ingredient needed to make 9 quarts of mixture J.</p>
<p>7. Explain Andrea's method:</p> 	<p>8. Use Andrea's method to find the amount of each ingredient needed to make 72 ounces of mixture B.</p>

## PRACTICE 2

Use tape diagrams to solve these problems.

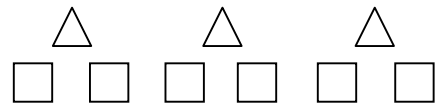
1. Milo likes to make fruit soda when he has people over to his house. He has determined that the juice to sparkling water ratio should be 4 : 3. He estimates that he will want 70 cups in all. How much juice and how much sparkling water will he need for the 70 cups?
  
  
  
  
  
  
  
  
  
  
2. Kendra makes tie-dyed shirts. Her most frequently used colors are orange and green.
  - a. For the orange dye, she uses red and yellow in a ratio of 3 : 2. How many quarts of red and yellow dye will she need if she wants to make 80 quarts of orange dye?
  
  
  
  
  
  
  
  
  
  
  - b. For the green dye, she uses blue and yellow in a ratio of 5 : 2. How many quarts of blue and yellow dye will she need if she wants to make 56 quarts of green dye?

# UNIT RATES

<b>Summary</b>	<b>Goals</b>
<p>We will explore the relationship between ratios and rates. We will define unit rate. We will represent ratios and rates with tables and double number line diagrams, and solve problems using rates.</p>	<ul style="list-style-type: none"> <li>Relate unit rate to ratio.</li> <li>Represent rates using symbols, words, tables, and double number line diagrams.</li> <li>Solve problems using rates, tables and double number line diagrams.</li> </ul>

### Warmup

Joann described the diagram to the right in eight different ways. Circle her statements below that are true.



1. There is a ratio of 3 triangles to 6 squares.      2. There is 1 triangle for every 2 squares.

3. There is 1 square for every  $\frac{1}{2}$  triangle.

4. If I double the number of squares to keep the triangle to square ratio the same, I would also have to double the number of triangles.

The black strip tells us we are starting the second lesson. Summary, Goals, and Warmup always kick off the start of a new lesson.

5. If I add 3 more triangles, I also have to add 3 more squares to keep the triangle to square ratio the same.

6. The ratio of squares to total number of shapes is 6 : 3.

7. Triangles represent  $\frac{3}{6}$  of the shapes.

8. Triangles represent  $\frac{1}{3}$  of the shapes.

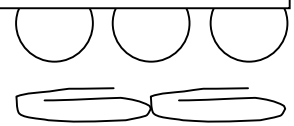
## UNIT RATES ASSOCIATED WITH RATIOS

The unit rate associated with the ratio  $a : b$ ,  $b \neq 0$ , is the number  $\frac{a}{b}$ , generally with units of “something per something” attached. The quotient  $\frac{a}{b}$  is also referred to as the value of a ratio.

Example: If you drive 40 miles in 2 hours, then your unit rate is  $\frac{40}{2} = 20$  miles per hour.

Now unit rate and the value of a ratio are defined.

1. The ratio of coins to paperclips in the picture to the right is 3 : 2.  
The unit rate can be written  $\frac{3}{2}$  coins per paperclip.



- a. The ratio of coins per paperclip can be written 


 to 1.

- b. Write the unit rate as a mixed number and as a decimal. \_\_\_\_\_

- c. Avi says, “There is no such thing as 1.5 coins.” Explain to Avi what this unit rate means using words and pictures.

2. Elyse added one more coin and one more paperclip so that the ratio of coins to paperclips changed. Write the new unit rate of coins per paperclip to prove Elyse’s addition changed the initial ratio.



## RATES AND TABLES

1. Here is mileage information for three different vehicles. Use Table I below to complete Table II below.

<b>TABLE I</b>	<b>Scooter</b>	<b>Bus</b>	<b>Train</b>
<b># of miles</b>	100	280	310
<b># of hours</b>	25	7	5

<b>TABLE II</b>	<b>Miles to Hours Ratio</b>	<b>Unit Rate (in miles per hour)</b>
<b>Scooter</b>	_____ miles in _____ hours	_____
<b>Bus</b>		
<b>Train</b>		

2. Explain why the ratios of miles to hours for the three vehicles are not equivalent.

3. The Bead Store sells beads in bags. Use Table III below to complete Table IV below.

<b>TABLE III</b>	<b>Bag A</b>	<b>Bag B</b>	<b>Bag C</b>	<b>Bag D</b>	<b>Bag E</b>
<b>Price of bag</b>	\$5.00	\$2.50	\$10.00	\$7.50	\$12.50
<b># of beads in the bag</b>	50	25	100	75	125

<b>TABLE IV</b>	<b>Bag A</b>	<b>Bag B</b>	<b>Bag C</b>	<b>Bag D</b>	<b>Bag E</b>
<b>Price per bead</b>	\$0.10				

4. Explain how you know that the ratio of the price per bag to the number of beads in a bag is equivalent for all the bags.

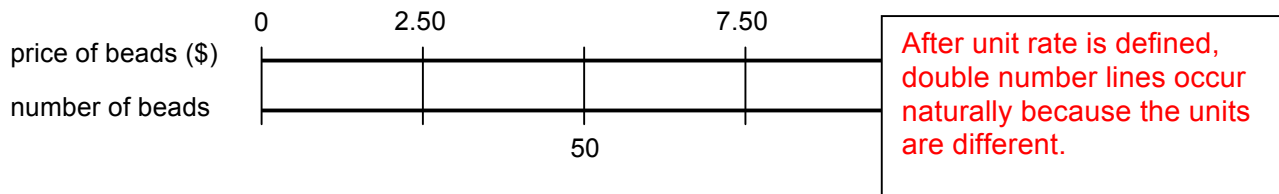
A unit price is a price for one unit of measure.

Example: In problem 3 above, the unit price for beads in each of the bags is \$0.10 per bead.

## RATIOS AND DOUBLE NUMBER LINES

A double number line diagram is a graphical representation of two variables in which the corresponding values are placed on two parallel number lines for easy comparison. Double number lines are often used when the two quantities have different units.

1. Refer to Table III on page 11 to make a double number line diagram.



2. How are the quantities on the double number line above organized differently from those in Table III on page 11?
3. Alberto jogs 5 yards every 2 seconds.

a. Complete the table below.

<b># of yards</b>	5			100			1	2
<b># of seconds</b>		4	8		60	1		

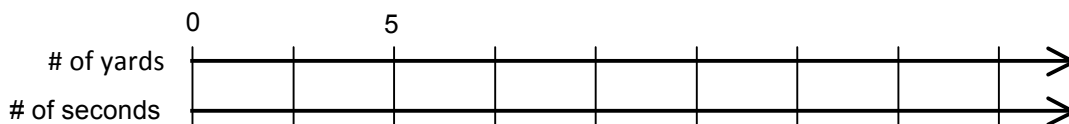
b. Complete each sentence.

Alberto's jogging rate is \_\_\_\_\_ yards for one second.

Alberto's unit rate in yards per second is \_\_\_\_\_.

At this rate, Alberto will jog \_\_\_\_\_ in one minute.

- c. Make a double number line diagram to represent this relationship. You do not need to put every value in the table on the lines.



## RATIOS AND DOUBLE NUMBER LINES (Continued)

4. Ling makes 5 bracelets every 4 hours.

a. Complete the table below.

	5		25	55	90		1	2
	4	8				1		

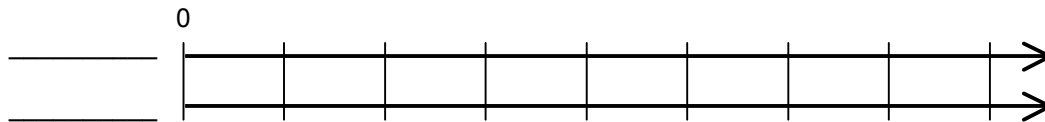
b. Fill in the blanks:

Ling’s rate of making bracelets is \_\_\_\_\_ for one hour.

Ling’s unit rate in bracelets per hour is \_\_\_\_\_.

At this rate, Ling will make \_\_\_\_\_ in 10 hours.

c. Make a double number line diagram to represent this relationship. You do not need to put every value in the table on the lines.



5. Now look at Table I on page 11. Castro says that the collection of traveling rates for the vehicles listed in the table are not equivalent. Therefore, Castro concludes that he cannot record this data on a double number line diagram. Use numbers and words to defend Castro’s conclusion.

## POSTER PROBLEMS

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D. I am group member \_\_\_\_\_.
- Each group will start at a numbered poster. Our group start poster is \_\_\_\_\_.
- Each group will have a different color marker. Our group marker is \_\_\_\_\_.

Part 2: Answer the problems on the posters by following your teacher's directions.

Part 3: Return to your seats.

Each semester one or more "Poster Problems."

This page gives some directions (more directions are in TP11 in Teacher Guide) and serves to hold each student individually accountable for the group practice activity.

Refer to the poster problems. Discuss and answer each question. Each answer has a "twist."

1. Dee Dee can buy 12 markers for \$7.20. How many markers can you buy for \$14.40?
2. A baseball team plays 8 games in 3 weeks. Devon says, "The games per week unit rate does not make sense." Explain what you think Devon means.
3. A recipe has a ratio of 3 cups of milk to 4 cups of flour. Elisa says combining 3 cups of milk and 4 cups of flour makes a 7 cup mixture. Margit disagrees. Who do you think is correct? Explain.
4. Danisha grew 2 inches in 40 weeks. Explain why this situation could make sense for Danisha, but not for other people.



# PRACTICE

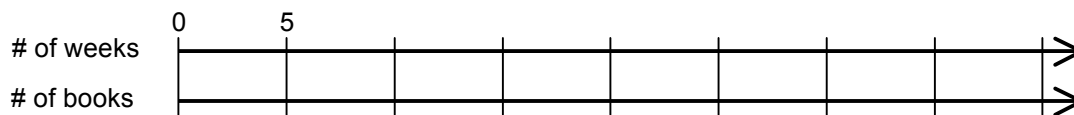
1. The two rate statements below are not the same. Use the concept of unit rates to explain the difference.

a. LaVerne earns \$10 in 2 hours.

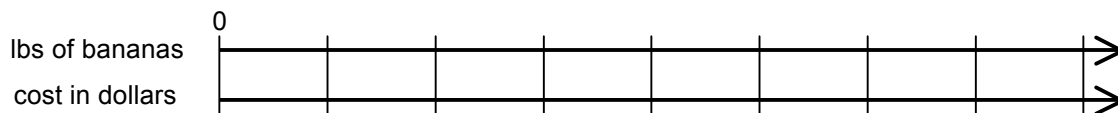
b. Shirley earns \$2 in 10 hours.

Complete the double number line diagrams below and use the results to answer each question in numbers and words.

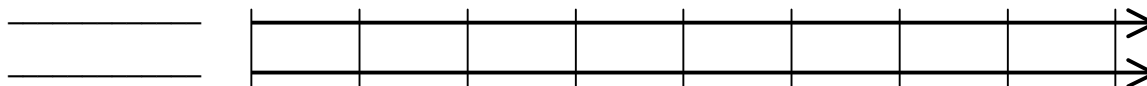
2. It takes Joey 5 weeks to read 4 books. At that rate, how many books can he read in 25 weeks?



3. Tommy can buy 2 pounds of bananas for \$0.80. At that rate, how much would it cost him to buy 16 pounds (lbs) of bananas?



4. Zippy can run 2 miles in 16 minutes. At that rate, how fast can he run 13 miles?



**PRACTICE (Continued)**

Assume constant rates for each problem.

5. You pay \$60 for 10 sandwiches.
  - a. What is the price per sandwich?
  - b. What is the price for 20 sandwiches at that rate?
  
6. You read 5 books in 2 weeks.
  - a. How many books do you read per week?
  - b. At that rate how many books would you read in 8 weeks?
  
7. A work day is 8 hours. You earn \$48 for one work day.
  - a. What is the hourly pay rate?
  - b. At that rate how much would you earn in 5 hours?
  
8. A small fruit basket contains 5 apples, 3 bananas, and 4 oranges.
  - a. What is the ratio of bananas to fruit in the basket?
  - b. A larger basket that maintains this ratio of bananas to fruit contains 48 pieces of fruit. How many bananas are in the basket?
  
9. You eat 2 pounds of potatoes every 8 weeks.
  - a. How many pounds of potatoes do you eat per week?
  - b. At that rate how many pounds of potatoes would you eat in 6 weeks?

# RATIO AND UNIT RATE PROBLEMS

## Summary

We will solve ratio and rate problems using tables, diagrams, and equations.

## Goals

- Solve ratio and rate problems using a variety of strategies.

## Warmup

Use tables, diagrams, or unit rates, as appropriate, to solve these problems.

Graham is paid \$90 for 5 hours of work.

The Warmup in this last lesson draws knowledge developed in the first two lessons.

1. At this rate, how much does Graham make for 8 hours of work?
2. At this rate, how long does Graham have to work to make \$63?
3. Graham makes lemonade at work by mixing 3 parts water and 2 parts lemon juice. How much water and lemon juice does he need to make 85 cups of lemonade?

# THE GREEN GROCER

<p><b>Green Grocer</b>  <i>Fruit Special of the Day</i></p> <p>2 melons for \$3.50</p>	<p style="color: red;">This lesson includes problems where students apply and extend the basic concepts they just learned. TP11 provides suggestions for implementation, which include doing some problems in groups, some using a jigsaw setup, and others individually.</p>
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1. What is the cost per melon? \_\_\_\_\_
2. Make a table below to show the cost in dollars of different numbers of melons.

<b>Cost of melons in dollars (<i>D</i>)</b>	3.50		7.00
<b># of melons (<i>M</i>)</b>	2	6	

3. Make a double number line diagram to show the cost in dollars of different numbers of melons.

4. Write an equation to compute the cost in dollars (*D*) of any number of melons (*M*).

$D = \underline{\hspace{2cm}}$

5. Circle numbers that represent the unit rate in problems 1, 2, 3, and 4 above.

6. How much will 42 melons cost? Explain.

7. How many melons can be purchased for \$84? Explain.

## THE TOOTHPASTE PROBLEM

Pippy observed that she used 3 tubes of toothpaste in 5 months. At that rate, she wondered how many tubes she would use in a year. Her friends were curious about this too, and they wanted to help her figure this out. Their work is started below.

1. Pippy began the table below. Fill in the missing information and explain this strategy.

This problem requires students to think more about using tables and scalar multiples to solve a problem.

		not enough	too much			
<b># of tubes of toothpaste</b>	3	6	9			
<b># of months</b>	5	10	15	1	2	12

2. Tippy made the shorter table below. Fill in the missing information and explain her reasoning based on this table.

		$\div 5$	$\times 2$	$\times 6$
<b># of tubes of toothpaste</b>	3			
<b># of months</b>	5	1	2	12
		$\div 5$	$\times 2$	$\times 6$

3. Zippy likes to do things quickly. Fill in the missing information in the table below and explain his reasoning.

		$\times \frac{12}{5}$
<b># of tubes of toothpaste</b>	3	
<b># of months</b>	5	12
		$\times \frac{12}{5}$

4. How much toothpaste will Tippy use in a year?

# APPLES, APPLES, APPLES

1. You see ads for apples at six different stores. Determine the best buy(s). Show all your work.

<p><u>Store A:</u> Apples cost \$2.00/lb.</p>	<p><u>Store B:</u> Apples come in a 5 lb bag for \$10.50.</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px; color: red;"> <p>Multiple strategies are needed to make comparisons on the next couple of pages.</p> </div>
<p><u>Store C:</u> Apples come in a 1.75 lb bag for \$3.60.</p>	<p><u>Store D:</u> Apples come in a 7.5 lb bag for \$14.25.</p>
<p><u>Store E:</u> Apples cost \$2.25 per pound with a coupon for \$0.12 off per half-pound.</p>	<p><u>Store F:</u> Apples cost \$2.25 per pound with a coupon for \$0.07 off per quarter-pound.</p>

2. You want to buy 5 pounds of apples. Explain which store you would choose.

3. Use your understanding of the concept of unit price to interpret each situation.

<p><u>Store G:</u> \$0.00 for 3 apples</p>	<p><u>Store H:</u> \$3 for 0 apples</p>
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## BUILDING A DECK

Ben is building a wooden deck that requires 400 feet of lumber. He called several lumber yards to find the best price and wrote down the information below.

<u>Store A</u> : The lumber costs \$2.50/ft.	<u>Store B</u> : Eight feet of lumber costs \$18.
<u>Store C</u> : The total for all 400 feet is \$910.	<u>Store D</u> : The total for the first 200 feet is \$500. After that, the cost is \$1.90/ft.
<u>Store E</u> : The first 300 feet cost \$2.80/ft. After that, the rate is \$0.75/ft.	

Show your work above and explain your reasoning below.

1. Which store provides Ben with the best buy to purchase 400 feet of lumber?

How much will this cost Ben?

2. If you worked at a store and wanted to make Ben a better deal, what unit price (per foot) could you offer him for your lumber?

## THE GRAIN GROCER

The Grain Grocer sells rice in bulk.  
The special of the day is to the right.

**Grain Grocer**

*Special of the Day*

Antonio said, "The ratio of the number of dollars to the number of pounds is 4:5. That's \$0.80 per pound."

Bianca said, "The sign means that the ratio of the number of pounds to the number of dollars is 5:4. That's 1.25 pounds per dollar."

1. Are Antonio and Bianca both correct? Explain.
2. Carlos needs two pounds of rice to make a casserole. Explain to Carlos how much money he will need.
3. Dora has \$10 and wants to stock up on rice. Explain to Dora how many pounds of rice she can buy.
4. Do you prefer to answer problems 2 and 3 using Antonio's rate of \$0.80 per pound, using Bianca's rate of 1.25 pounds per dollar, or using another strategy? Explain.

The next couple of pages tie many of the concepts developed in this packet together. Students might use a double number line here or some other strategy or representation of their choice.



## THE ASSEMBLY

The school auditorium has 330 seats. When the 6<sup>th</sup> grade students went to assembly period 1, seats were filled at a ratio of 9 occupied to 2 unoccupied. When the 7<sup>th</sup> grade students went to assembly period 2, seats were filled at a ratio of 5 occupied to 1 unoccupied.

1. How many 6<sup>th</sup> grade students went to the assembly?

This problem might be solved using a tape diagram or some other strategy or representation of their choice.

2. How many 7<sup>th</sup> grade students went to the assembly?

3. If 300 8<sup>th</sup> grade students went to assembly period 3, what was the ratio of occupied to unoccupied seats?

## THE PAINT MISTAKE

Jimbo's daughter says she wants to paint her room pink. He thinks the color can be made by using 2 scoops of white paint for every 3 scoops of red paint. When Jimbo makes a sample using exactly 5 scoops, his daughter says, "No, you were supposed to use 3 scoops of white paint for every 2 scoops of red paint."

Jimbo does not want to waste any paint. Without throwing out the 5 scoop mixture he already made, describe how Jimbo can correct the mistake when making a larger batch of paint the way his daughter wants it.

# SKILL BUILDERS, VOCABULARY, AND REVIEW

## SKILL BUILDER 1

Compute the following sums and differences.

1. $\frac{1}{4} + \frac{1}{3}$	2. $\frac{3}{5} - \frac{1}{10}$	3. $\frac{3}{4} + \frac{5}{6}$	4. $4\frac{3}{4} - 4\frac{1}{2}$
<div style="border: 1px solid black; padding: 5px; color: red; margin: 0 auto; width: 80%;">                     Skill Builders usually have 3-4 pages of review at the start, strategically built in, both to include review for fluency's sake, and to support learning in the current or an upcoming packet.                 </div>			

5. Estimate the location of each number on the number line below.

- $\frac{7}{7}$        $1\frac{2}{3}$        $\frac{6}{5}$        $\frac{2}{3}$        $\frac{4}{5}$        $\frac{4}{3}$        $1\frac{1}{5}$        $\frac{2}{3}$        $\frac{1}{12}$



6. Which of the numbers from problem 5 above are equal? \_\_\_\_\_

7. Javier incorrectly thinks that 0.3 and  $\frac{1}{3}$  are equal. Explain to Javier why they are not.

Rewrite each number below in its equivalent representations.

	Fraction or mixed number	Decimal	Percent
8.	$\frac{1}{8}$		
9.		0.02	
10.			18.9%
11.		1.06	

## SKILL BUILDER 2

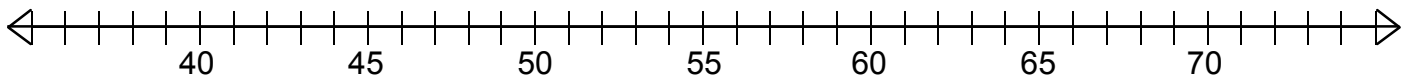
The data in the table below represents the number of division problems individual students answered correctly in two minutes.

38	42	46	46	38	43	39	74	36	46	35	40
66	45	49	45	46	46	46	73	52	50	42	45

1. Arrange the data from above in numerical order in the table below.

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

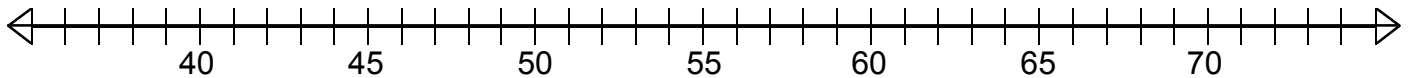
2. Make a dot plot for the data using the number line below.



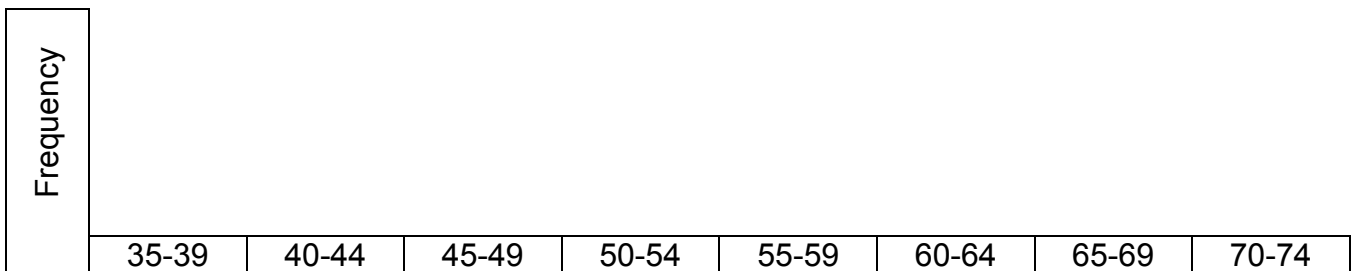
3. Find the five-number summary for the data.

( \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_ )  
 Min                      Q<sub>1</sub>                      med (Q<sub>2</sub>)                      Q<sub>3</sub>                      Max

4. Construct a box plot below for this data.



5. Construct a histogram below for this data.



6. Does the data skew left or right? \_\_\_\_\_. What does this say about the number of division problems the class can do in 2 minutes?

## SKILL BUILDER 3

Simplify each expression first if possible. Then complete the table below based on the simplified expressions.

Expression	Number of terms	Constant term(s)	Term(s) with Variables	Coefficient of the variable(s)
1. $6 + 5m - 2$				
2. $w + w + w + w + w + w$				

Evaluate each expression for  $w = 5$ ,  $x = 2$  and  $y = 3$ .

3. $w + (x + y)^3$	4. $(xy - w)^7$
--------------------	-----------------

Use exactly one 3, one 5, and one 2, along with any mathematical operations and symbols, to create expressions that meet the following conditions.

5. The target number is greater than 100.	6. The target number is even and greater than 10.	7. The target number is a prime number.
---	---	---

Solve using any strategy. Write MM if mental math is used. Otherwise show your work.

8. $x - 3.5 = 7.05$	9. $y + 1.7 = 2.1$	10. $0.9y = 9.9$
---------------------	--------------------	------------------


Compute.

11. $1092 \div 12$	12. $52.38 \div 5.4$
--------------------	----------------------

### SKILL BUILDER 4

1. Explain in your own words what this statement means: the ratio of the number of blue marbles to the number of red marbles in a bag is 2 to 3.

First page of practice of current work appears here.

Here are some arrows and stars: 

Write each ratio.

2. The number of arrows to the number of stars	___ to ___	___ : ___
3. The number of stars to the number of arrows	___ to ___	___ : ___
4. The number of stars to the total number of shapes	___ to ___	___ : ___
5. The total number of shapes to the number of arrows	___ to ___	___ : ___

6. Charlie adds six more stars to the collection of shapes above. How many arrows would he need to add to keep the ratio of arrows to stars the same? Explain.

7. The ratio of women to men in a coed volleyball tournament is 2 to 3.

a. Create a table below for this situation.

<b>Total</b>					

- b. Using the columns in the table above, write two different ratios and explain how you know these ratios involving numbers of people in the tournament are equivalent. Use an arrow diagram if desired.

## SKILL BUILDER 5

Franco likes to make a lemon tea that is in a ratio of 4 cups of lemonade to 5 cups of iced tea. He is hosting a party for 15 people and estimates that each person will drink 3 cups of lemon tea.

1. How much lemon tea should he make so every guest has 3 cups?
2. Construct a tape diagram to figure out how many cups of lemonade and iced tea Franco should use.

3. How many cups of lemonade should Franco put in the lemon tea?
4. How many cups of iced tea should Franco put in the lemon tea?

To help him make different amounts of lemon tea, Franco started the table below.

5. Complete the table.

<b>lemonade (# of cups)</b>	4			16	
<b>iced tea (# of cups)</b>	5		15		
<b>Total (# of cups)</b>		18			144

6. Which of the recipes below could be used to make Franco's lemon tea?

a. 2 cups of lemonade 2.5 cups of iced tea	b. 5 cups of lemonade 4 cups of iced tea
c. 20 cups of lemonade 25 cups of iced tea	d. $1\frac{1}{4}$ cups of iced tea 1 cup of lemonade

## SKILL BUILDER 6

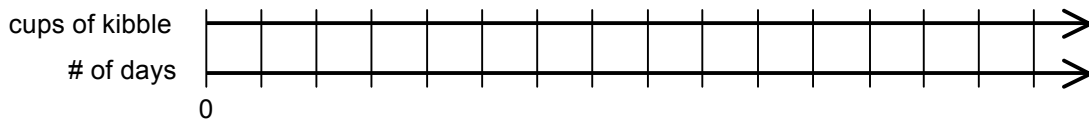
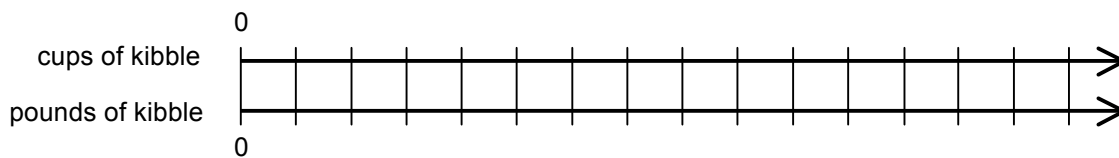
1. Give an example of a rate.

2. You work 10 hours and make \$118. Complete the table below. Then circle the column in the table that could be used to represent your hourly pay rate (unit rate).

<b>pay (\$)</b>	118	236		1180				
<b># of hours</b>	10		2		100	1	5	25

An 8 pound bag of dog kibble contains about 26 cups of kibble. Barry’s dog Cocoa eats  $2\frac{1}{2}$  cups of kibble per day. Use this information to complete problems 3-6.

3. Make double number lines below to represent these ratios.



4. How many cups of kibble does Cocoa eat in two weeks?

5. The table to the right shows the cost of kibble in various-size bags. Do the costs per pound for the bags represent equivalent ratios? \_\_\_\_\_ Explain.

<b>Bag size (in pounds)</b>	<b>Cost (\$)</b>
4	6.00
8	10.00
20	20.00

6. If Barry wants to buy Kibble for Cocoa that will last for two weeks, what should he purchase?



**SKILL BUILDER 7**

1. A shade of pink paint requires 5 parts of red paint and 2 parts of white paint. Use a table or diagram for the following.
  - a. How much red paint should be used if you have 1 gallon of white paint?
  
  
  
  
  
  
  
  
  
  
  - b. How much white paint should you use if you have 25 gallons of red paint?
  
2. Miko wants to make a whiter shade of pink paint. Write two ratios (red paint to white paint) that would make a whiter shade of pink compared to the mixture in problem 1.  
Explain your reasoning.
  
3. Stephen wants to make a redder (darker) shade of pink paint. Write two ratios (red paint to white paint) that would make a redder (darker) shade of pink compared to the mixture in problem 1.  
Explain your reasoning.

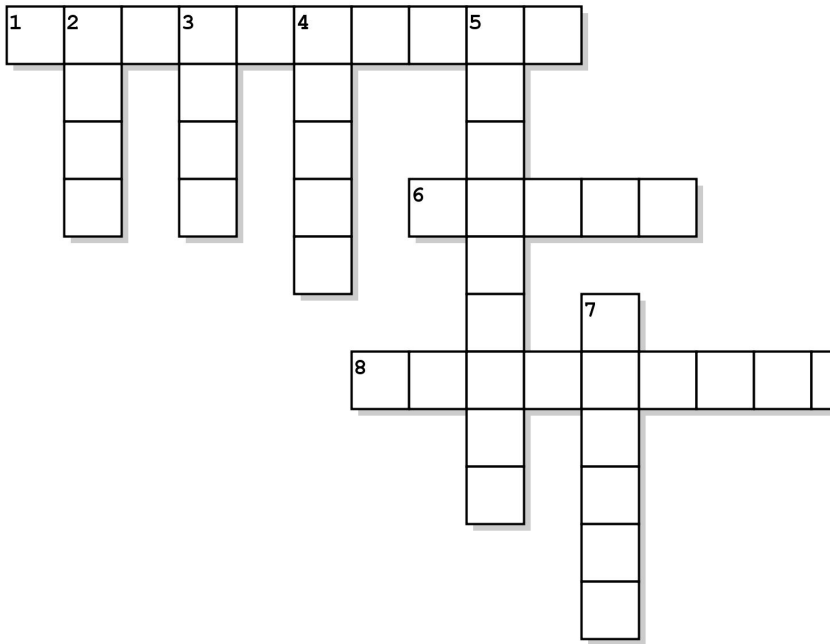
DeSean and Danny both wash cars on weekends to earn extra money. DeSean washes at a rate of 5 cars per every 2 hours. Danny washes 7 cars per every 3 hours.

4. Who washes cars at a faster rate?
  
  
  
  
  
  
  
  
  
  
5. DeSean hurts his arm, so for the next several weekends he is washing 5 cars per every 2.5 hours. Use a table or diagram and explain how his injury changes the answer above.

**SKILL BUILDER 8**

1. Hugo is buying cupcakes for a birthday party. At Cupcake Land, he can buy 6 cupcakes for \$5. At Cake-in-a-Cup, he can buy 9 similar cupcakes for \$8. Which store offers Hugo a better buy? Explain.
2. Harvey says that The Sweet Shop, offering 12 cupcakes for \$10, is a better deal than the choices in problem 1 because he can get the most cupcakes. Critique Harvey's reasoning.
3. Minuet gets \$30 in allowance every two weeks. She is saving all of her allowance to buy a prom dress that costs \$210. How many weeks will it take her to save up enough money for the dress? Use a table, tape diagram, or double number line to show your reasoning.
4. Lexie is making an orange punch drink for a party. She puts 4 pints of orange drink and 2 pints of water into a punch bowl. Her friend Olivia says, "Oh no, you were supposed to make a mixture that is 3 parts orange drink and 4 parts water." How can Lexie add to the current punch bowl mixture and correct the mistake so that the orange drink mixture is done the way Olivia suggests.

## FOCUS ON VOCABULARY



After Skill Builders comes Focus on Vocabulary. Most of the words used here can be found in the Word Bank in the front of this SP.

This page is always followed by Selected Response, Knowledge Check, and Home-School Connection.

**Across**

- 1 5 is a \_\_\_ used to show that 2:3 and 15:15 are equivalent.
  
- 6 a pair of numbers in a specific order
  
- 8 Equivalent ratios are examples of this kind of relationship

**Down**

- 2 the value of a ratio: \_\_\_\_\_ rate
  
- 3 A(n) \_\_\_ diagram shows equivalent ratios when units are the same.
  
- 4 \$3 per 1 melon is an example of a unit \_\_\_\_\_.
  
- 5  $D = 1.75M$  and  $3x + 6 = 9$  are examples of \_\_\_\_\_.
  
- 7 A(n) \_\_\_\_\_ number line shows equivalent ratios when units are different.

(For word hints, see the word bank and other vocabulary used in this packet.)

## SELECTED RESPONSE

Show your work on a separate sheet of paper and select the best answer(s).

1. Mrs. Robert's math class has 13 boys and 11 girls. Which of the following shows the relationship of the number of girls to the total number of students in the class? Choose ALL that apply.

A. 13 : 11                      B. 11 : 13                      C. 11 : 24                      D. 24 : 13

For problems 2 and 3, use the collection of shapes to the right:



2. Which of the following represent the ratio of triangles to circles? Choose ALL that apply.

A. 3 to 2                      B. 2 to 3                      C. 3 to 5                      D. 1.5 to 1

3. Carlos added 4 more circles and some triangles to the collection of shapes above. The ratio of triangles to circles stayed the same. How many triangles did Carlos draw?

A. 3                      B. 4                      C. 5

Exercises where more than one answer may be correct help to prepare students for these types of items on SBAC or PARCC.

For problems 4-6: Jacqueline worked 40 hours and made \$480.

4. What was Jacqueline's hourly pay rate? Choose ALL that apply.

A.  $\frac{480}{40}$  dollars per hr                      B.  $\$ \frac{480}{40}$  per hr  
 C. \$12 per hr                      D.  $\$ \frac{1}{12}$  per hr

5. At the rate stated in problem 4, how much would Jacqueline expect to make in 160 hours?

A. \$120                      B. \$192                      C. \$960                      D. \$1,920

6. At the rate stated in problem 4, how many hours would Jacqueline work to make \$288?

A. 12                      B. 20                      C. 24                      D. 240

7. Jonette knows that the ratio of blue marbles to red marbles in a bag is 3 : 4. How many blue and red marbles could be in the bag? Choose all that apply.

A. 5 blue, 6 red                      B. 6 blue, 8 red  
 C. 27 red, 36 blue                      D. 27 blue, 36 red

## KNOWLEDGE CHECK

Show your work on a separate sheet of paper and write your answers on this page.

### 11.1 Ratios

Here are some stars and arrows: 

1. What is the ratio of the number of stars to the total number of shapes?
2. Use words to describe what the ratio  $5 : 9$  means in this context.
3. If George added 18 shapes to the collection but the ratio of stars to arrows remains the same, how many stars did he add?

These problems are somewhat representative of those in each lesson, and may be used for review or formative assessment.

### 11.2 Unit Rates

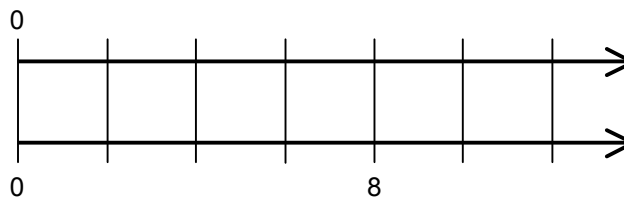
A horse eats 132 bales of hay in 12 months. Answer the following questions and use a tape diagram to show your reasoning.

4. How many bales of hay does the horse eat per month?
5. At that rate, how many bales of hay would the horse eat in 8 months?
6. How much hay would 3 horses eat in 12 months?
7. How long would 198 bales last if it were used to feed one horse?

### 11.3 Ratio and Unit Rate Problems

Maria earns the same amount each day for her part time job. After working 8 days she made \$1,200.

8. Use a double number line diagram to find out how much she made in 6 days.



9. How much will she make in 40 days? Show your work.

## HOME-SCHOOL CONNECTION

Here are some problems to review with your young mathematician.

About 1 out of 10 people are left-handed.

1. Make a tape diagram to represent this situation.
2. In a group of 70 people, about how many people would you expect to be left-handed? Explain how to solve this problem using the tape diagram.

The intent of this page is to provide an opportunity for students to explain to parents or guardians what they are learning.

At this time parents or guardians might check to see if students are completing their work in the packet.

Raymond walked in a fundraiser for cancer research. He walked 18 miles in 6 hours and raised \$1,440.

3. How many dollars per hour walking did he raise?
4. Create a double number line diagram to show the relationship between the number of miles walked and the number of hours walked.
5. Use your double number line to determine how long it took for him to walk 12 miles.

Parent (or Guardian) Signature \_\_\_\_\_

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On occasion, some packets will have blank pages following the Home-School Connection until you reach the Common Core State Standards – Mathematics on the back cover.

# COMMON CORE STATE STANDARDS – MATHEMATICS

## STANDARDS FOR MATHEMATICAL CONTENT

<b>6.RP.A</b>	<b>Understand ratio concepts and use ratio reasoning to solve problems.</b>
6.RP.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. <i>For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”</i>
6.RP.2	Understand the concept of a unit rate $a/b$ associated with a ratio $a:b$ with $b \neq 0$ , and use rate language in the context of a ratio relationship. <i>For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is <math>3/4</math> cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”</i>
6.RP.3a	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations: Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and <del>plot the pairs of values on the coordinate plane</del> . Use tables to compare ratios.
6.RP.3b	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations: Solve unit rate problems including those involving unit pricing and constant speed. <i>For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?</i>

## STANDARDS FOR MATHEMATICAL PRACTICE

MP1	Make sense of problems and persevere in solving them.
MP3	Construct viable arguments and critique the reasoning of others.
MP4	Model with mathematics.
MP7	Look for and make use of structure.

These are the major content standards that are addressed in this packet. It is common for a standard to fully play out over multiple lessons and multiple packets. Note that the “cross-out” above means that the phrase was not included in this packet. It is addressed elsewhere in the program.

The practice standards above are not an exhaustive list, but are good examples of MPs in these lessons (See Teacher Note 1 in TP11). All of the MPs are revisited frequently throughout each course.



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