

# WHOLE NUMBERS: USING AN AREA MODEL TO EXPLAIN MULTIPLICATION

The inability to recall arithmetic facts and perform multiplication fluently is often cited by teachers as a significant barrier to students' mathematics performance. This lesson gives students some strategies for multiplication. They learn to derive more difficult facts from easier facts, and they perform multiplication using a visual "area model." The key feature of the area model is that it represents the product of two numbers as a rectangular region made up of unit squares. The area of a rectangle is the number of unit squares that make up the rectangle.

This lesson connects to algebra and algebraic thinking in various ways. It uses multiple representations to explain the process of multiplication. It highlights the commutative and distributive properties, which are fundamental to the real number system and to success in manipulating algebraic symbols. The area model for multiplication establishes the groundwork for helping visual learners in the conceptual understanding of the traditional algebraic skills of polynomial multiplication and factoring.

<p><b>Math Goals</b></p> <p>(Standards for posting in <b>bold</b>)</p>	<ul style="list-style-type: none"> <li>• Use logical reasoning to derive multiplication facts. (Gr2 NS3.1; Gr7 MR1.3; Gr7 MR2.2)</li> <li>• Use the associative and commutative properties of multiplication. (Gr3 AF1.5; Gr7 AF1.3)</li> <li>• Use the distributive property. (Gr5 AF1.3; Gr7 AF1.3)</li> <li>• Use expanded notation. (Gr3 NS1.5)</li> <li>• Compare traditional algorithms for multiplication to area models for multiplication. (Gr4 NS3.2; Gr7 MR1.3; Gr7 MR2.2)</li> </ul>
<p><b>Summative Assessment</b></p>	<p>Future Week</p> <ul style="list-style-type: none"> <li>• <b>Week 4: Whole Numbers: Multiplication, Properties of Multiplication</b> (Gr2 NS3.1; Gr3 AF1.5; Gr4 NS3.2; Gr5 AF1.3)</li> </ul>

## 1.1 Whole Numbers: Using an Area Model to Explain Multiplication

### PLANNING INFORMATION

Estimated Time: 90 – 120 Minutes

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<p><b>Student Pages</b></p> <p>* SP1: Ready, Set, Go            * SP2: Properties of Multiplication            SP3: Multiplication Made Easy            * SP4: Multiplication Using an Area Model            SP5: Multiplication Using an Area Model (continued)            * SP6: Area Problems            SP7: Area Problems (continued)</p>	<p><b>Materials</b></p> <p>Square tiles (optional)            Overhead base-10 blocks</p>	<p><b>Reproducibles</b></p> <p>* R1: Base-10 Blocks</p>
<p><b>Homework</b></p> <p>SP7: Area Problems (continued)</p> <p>Find each product with an area model. Check using traditional algorithm:</p> <p>9 X 8            75 X 4            213 X 66            1,024 X 370</p>	<p><b>Prepare Ahead</b></p> <p>Become familiar with area models for explaining multiplication.</p>	<p><b>Management Reminders</b></p> <p>To encourage discussion, have groups work on different problems in class, and then convince the class of their solutions by sharing at the overhead.</p>
<p><b>Assessment</b></p> <p>* SP21: Knowledge Check 1            R2 Knowledge Challenge 1            A5-6: Weekly Quiz 1</p>	<p><b>Strategies for English Language Learners</b></p> <p>Formal definitions of mathematical properties will be difficult for some students. Start with examples of the properties in action and work from there towards a class definition and the label. After this knowledge base is established, then move to the formal definition.</p>	<p><b>Strategies for Special Learners</b></p> <p>All students benefit from explaining their strategies in small groups and to the whole class.</p> <p>Making connections explicit from the blocks in the area model to a pictorial model and then the abstract multiplication algorithm is important for all special learners.</p>

\* Recommended transparency: Blackline masters for overheads 5-9 and 13 can be found in the Teacher Resource Binder.

## 1.1 Whole Numbers: Using an Area Model to Explain Multiplication

THE WORD BANK	
product	<p>A <u>product</u> is the result of multiplying two or more numbers. The numbers being multiplied to form the product are the <u>factors</u> of the product.</p> <p>Example: The product of 7 and 8 is 56, written <math>7 \cdot 8 = 56</math>. The numbers 7 and 8 are both factors of 56.</p>
area model	<p>The <u>area model</u> for multiplication is a pictorial way of representing multiplication. In the area model, the length and width of a rectangle represent factors, and the area of the rectangle represents their product.</p>
associative property of multiplication	<p>The <u>associative property of multiplication</u> states that <math>(a \cdot b) \cdot c = a \cdot (b \cdot c)</math> for any three numbers <math>a</math>, <math>b</math>, and <math>c</math>. In other words, the product does not depend on the grouping of the factors. We can therefore write the product unambiguously as <math>a \cdot b \cdot c</math>.</p> <p>Example: <math>(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5) = 3 \cdot 4 \cdot 5</math>.</p>
commutative property of multiplication	<p>The <u>commutative property of multiplication</u> states that <math>a \cdot b = b \cdot a</math> for any two numbers <math>a</math> and <math>b</math>. In other words, the product does not depend on the order of the factors.</p> <p>Example: <math>3 \cdot 5 = 5 \cdot 3</math></p>
distributive property	<p>The <u>distributive property</u> states that <math>a(b + c) = ab + ac</math> and <math>(b + c)a = ba + ca</math> for any three numbers <math>a</math>, <math>b</math>, and <math>c</math>.</p> <p>Examples: <math>3(4 + 5) = 3(4) + 3(5)</math> and <math>(4 + 5)8 = 4(8) + 5(8)</math>.</p>
standard form of a number	<p>The <u>standard form of a number</u> is the usual expression for the number, with one digit for each place value.</p> <p>Example: In standard form, four thousand nine is written 4,009.</p>
expanded form of a number	<p>An <u>expanded form of a number</u> is an expression for the number that shows explicitly the place value of each digit.</p> <p>Examples: <math>4,279 = (4 \times 10^3) + (2 \times 10^2) + (7 \times 10^1) + (9 \times 10^0)</math>  <math>4,279 = (4 \times 1,000) + (2 \times 100) + (7 \times 10) + (9 \times 1)</math>  <math>4,279 = 4,000 + 200 + 70 + 9</math></p>

## 1.1 Whole Numbers: Using an Area Model to Explain Multiplication

### MATH BACKGROUND


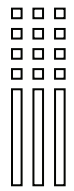
#### The Distributive Property

<p>Math Background 1 Introduce 1, 2</p>	<p>The <u>distributive property</u> (or <u>distributive law</u>) relates the operations of multiplication and addition. It guarantees that multiplication and addition are compatible. The term “distributive” arises because the law is used to distribute the factor outside the parentheses over the terms inside the parentheses. In the identity <math>3(4 + 5) = 3(4) + 3(5)</math>, the 3 is “distributed” over the 4 and the 5.</p>
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#### The Commutative and Associative Properties of Multiplication

<p>Math Background 2 Summarize 1 Introduce 2</p>	<p>The <u>commutative property of multiplication</u> tells us that we can write factors in any order:  <math display="block">3 \cdot 4 = 4 \cdot 3.</math> The <u>associative property of multiplication</u> tells us that to compute the product of say 3, 4, and 5, we can multiply first the 3 and 4, and then multiply the product by 5, or we can multiply the 4 and 5 first, and then multiply the product by 3, the result is the same:  <math display="block">(3 \cdot 4) \cdot 5 = 12 \cdot 5 = 60 \quad 3 \cdot (4 \cdot 5) = 3 \cdot 20 = 60</math> Since we arrive at the same result no matter how we group the factors, we can denote the product unambiguously by <math>3 \cdot 4 \cdot 5</math>.</p> <p>The associative and commutative properties of multiplication are also called the <u>associative and commutative laws</u> (or <u>rules</u>) of multiplication.</p>
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#### Does 3 X 14 Have the Same Value as 14 X 3?

<p>Math Background 3 Introduce 2 Extend 2 Summarize 2</p>	<p>Some elementary textbooks use a convention that implicitly connects language to order in multiplication. For example, “3 rows of 14” may be consistently written as “<math>3 \times 14</math>,” and accompanied by a diagram showing 3 rows and 14 objects in each row. This helps students model the multiplication problem, and the convention provides consistency for work and language. However, this convention is not based on mathematical properties, and it can lead to student misconceptions.</p> <div style="display: flex; align-items: center; justify-content: center; gap: 20px;"> <div style="text-align: center;">  <p><math>3 \times 14</math></p> </div> <div style="text-align: center;"> <p>3</p> <p><math>\times 14</math></p> </div> <div style="text-align: center;"> <p>Or</p> </div> <div style="text-align: center;"> <p>14</p> <p><math>\times 3</math></p> </div> <div style="text-align: center;">  <p><math>14 \times 3</math></p> </div> <div style="text-align: center;"> <p>14</p> <p><math>\times 3</math></p> </div> <div style="text-align: center;"> <p>Or</p> </div> <div style="text-align: center;"> <p>3</p> <p><math>\times 14</math></p> </div> </div> <p>The <u>commutative property</u> of multiplication asserts that the product does not depend on the order of the factors. For instance, each of the products <math>3 \times 14</math> and <math>14 \times 3</math> is equal to 42. Nonetheless, for some problems it is important to pay attention to context. Although both actions require 42 marbles, the filling of 3 bags with 14 marbles each will require different supplies than the filling of 14 bags with 3 marbles each. A piece of beachfront property (100 meters <math>\times</math> 30 meters) will be worth more money if the beachfront is 100 meters long rather than 30 meters long.</p> <p>Just as school textbooks consistently label rectangles by “length <math>\times</math> width,” so mathematicians label matrices as “rows <math>\times</math> columns.” An <math>m \times n</math> matrix always has <math>m</math> rows and <math>n</math> columns. The indices of the matrix entries are arranged by convention so that the first index represents the row and the second represents the column of the matrix entry.</p>
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## 1.1 Whole Numbers: Using an Area Model to Explain Multiplication

### TEACHING TIPS

#### Digit Name vs. Digit Value

<p>Teaching Tip 1</p> <p>Introduce 2 Explore 2</p>	<p>Stress place value in multiplication by distinguishing between the name of the digit and the value it stands for. The 2 in 24 stands for <math>2 \times 10 = 20</math>, not 2. Base-10 blocks and area model diagrams emphasize the value that each digit stands for because they use expanded notation to build the answer.</p>
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#### Drawing Rectangles for an Area Model

<p>Teaching Tip 2</p> <p>Introduce 2</p>	<p>The area model is an alternative and efficient way to multiply. Encourage students to draw rectangles, even though the rectangles may not be drawn to scale. If students need to use base-10 blocks as a transitional step, change the numbers in the problems to match the quantity of blocks that are available.</p>
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#### Using an Area Model to Record Multiplication

<p>Teaching Tip 3</p> <p>Introduce 2 Summarize 2 Practice</p>	<p><b><i>Is it okay to permit students to use the area model as a recording method for multiplication?</i></b> Yes. An area model not only helps to explain why the standard algorithm commonly taught in the United States for multiplication works, it is an efficient recording alternative. Some students (especially visual learners and those who have difficulty keeping numbers lined up in multiplication problems) may prefer it. Furthermore, this method has certain benefits. It illuminates important mathematical concepts (such as the distributive property), allows for computational flexibility (expanded notations allow students to use derived facts), and reinforces the concept of area. Finally, when students take algebra, they are likely to see the area model when they learn to multiply and factor polynomials.</p>
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#### Formal Algebra vs. Algebraic Thinking

<p>Teaching Tip 4</p> <p>Practice</p>	<p>Formal algebra and algebraic thinking are not synonymous. Formal algebra conjures up memories of the abstract, symbol-driven course that we took in school. Algebraic thinking starts much earlier. For instance, the California Mathematics Framework for California Public Schools includes algebra and functions at each grade level, starting in kindergarten.</p> <p>In this program, algebraic thinking is organized into two main categories (Kriegler, 2007):</p> <ul style="list-style-type: none"> <li>• Mathematical thinking tools, which include problem-solving skills, representation skills, and reasoning skills</li> <li>• Fundamental algebraic ideas, which include algebra as generalized arithmetic, algebra as a language for mathematics, and algebra as a tool for functions and mathematical modeling</li> </ul> <p>In this and selected future lessons, the role of algebraic thinking will be explained.</p>
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## 1.1 Whole Numbers: Using an Area Model to Explain Multiplication

### PREVIEW / WARMUP

Whole Class

➤ SP1\*  
Ready, Set, Go

- Introduce the goals and standards of the lesson. Discuss important vocabulary as relevant.
- Students complete the skip counting chart. Ask students to read each column in unison to check solutions.

***How are a skip counting list for 3s and multiplication by 3 related?***

The first number (3) represents “one group of 3” =  $1 \times 3$ . The second number (6) represents “2 groups of 3” =  $2 \times 3$ , etc.

***How can you use a skip counting list for 3s to find  $4 \times 3$ ?*** The fourth number on the list represents “four groups of 3.” ***How can you use it to find  $3 \times 4$ ?*** By the commutative property of multiplication, “three groups of 4” and “four groups of 3” give the same product. Therefore, the skip counting list for 3s also applies.

***How can you use a skip counting list to multiply  $7 \times 5$ ?*** Find the fifth number in the multiples of 7 list or the seventh number in the multiples of 5 list.  ***$6 \times 8$ ?*** Find the sixth number in the multiples of 8 list or the eighth number in the multiples of 6 list.  ***$3 \times 7$ ?*** Find the third number in the multiples of 7 list or the seventh number in the multiples of 3 list.

### INTRODUCE 1

Whole Class

➤ SP2  
Properties of  
Multiplication

Math Background 1

- Students read the properties of multiplication aloud and answer questions. Discuss as needed.
- Introduce the concept of “derived facts.”

***Last year, a student said she had difficulty remembering the fact for  $6 \times 7$ . But she did know  $3 \times 7 = 21$ . She added  $21 + 21 = 42$  in her head, and that’s how she “derived” the multiplication fact  $6 \times 7 = 42$ . Why does her method work?*** By the distributive property,  
 $6 \times 7 = (3 + 3)7 = 3 \cdot 7 + 3 \cdot 7 = 21 + 21 = 42$ .

## 1.1 Whole Numbers: Using an Area Model to Explain Multiplication

### EXPLORE 1

Pairs

➤ SP3  
Multiplication Made  
Easy (Derived  
Facts)

- Students select one of the multiplication facts that are typically difficult and record ways to remember them. Encourage students to share derived fact strategies with each other.
- Students complete multiplication by multiples of 10.

### SUMMARIZE 1

Whole Class

➤ SP3  
Multiplication Made  
Easy (Derived  
Facts)

- Record derived fact strategies for difficult facts on the board or on chart paper. Organize them according to the type of strategy used. Some examples are listed here.

Derived Fact Strategies				
Skip Count	Double	Halve	Add On	Take Away
4	$3 \times 7 = 21$ ,	$6 \times 10 = 60$ ,	$6 \times 3 = 18$	$10 \times 3 = 30$
8	$21 + 21 = 42$	So	Think 19, 20, 21	$30 - 3 = 27$
12	So	$6 \times 5 = 30$	so	So
16	$6 \times 7 = 42$	$5 \times 6 = 30$	$7 \times 3 = 21$	$9 \times 3 = 27$
	$7 \times 6 = 42$		$3 \times 7 = 21$	$3 \times 9 = 27$

Math Background 2

- Review strategies for multiplying by multiples of 10. Combine with fact recall strategies if needed.

**What do you think about when you multiply  $60 \times 8$ ?** Students will likely say they “multiply  $6 \times 8$  and add a zero.” This technique works, but offer an alternative to this sloppy, imprecise language because it may interfere with learning ( $6 \times 8 + 0 = 48$ ). A better way to state this is “multiply  $6 \times 8$  and **put** a zero at the end.”

**Why does this work?** Write 60 as the product of  $6 \times 10$ . Use the associative and commutative properties:  $(6 \times 10) \times 8 = (6 \times 8) \times 10 = 48 \times 10 = 480$ .

# 1.1 Whole Numbers: Using an Area Model to Explain Multiplication

## INTRODUCE 2

Whole Class

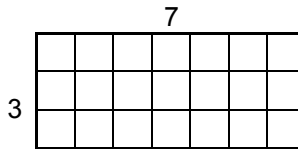
➤ SP4\*-5  
Multiplication Using  
an Area Model

Materials:  
Square-tiles  
Base-10 blocks

Math Background  
1,2,3

Teaching Tip 1,2,3

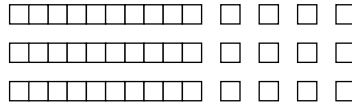
- Remind students of the meaning of the area of a figure (a measure of size expressed in square units), and how to find the area of a rectangle (multiply the length times the width).
- Use tiles or a diagram to make a rectangle that is  $3 \times 7$ . Break the rectangle into two parts to demonstrate that  $3 \times 7 = 3(5 + 2)$  and  $3 \times 7 = (1 + 2)7$ . Write in the partial products.



**What does the first rectangle show?** 3 rows of 7 = 21, 7 columns of 3 = 21,  $3 \times 7 = 21$ . This is an example of the commutative property of multiplication.

**Suppose we break the rectangle into 2 parts. What does this rectangle show?**  $3(2 + 5) = 21$ ,  $(3 \times 2) + (3 \times 5) = 21$ . This is an example of the distributive property.

- Use base-10 blocks to illustrate problem #4. Put  $3 \times 14$  on overhead as 3 rows of 14.



**What is the total value of this collection of base-ten blocks?** 42  
**How can we find the value without counting the blocks one by one?** A possible strategy: 3 groups of 10 + 3 groups of 4 =  $30 + 12 = 42$ .

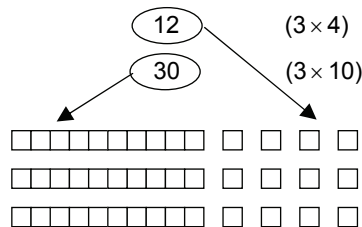
**What property is shown here?** The distributive property.

**How does this picture show the distributive property?**

$$3(10 + 4) = 3(10) + 3(4)$$

- Ask students to multiply  $3 \times 14$  using the traditional algorithm, but write out the partial products. Be sure students see the relationship between the partial products in the algorithm and the area model representation.

$$\begin{array}{r} 14 \\ \times 3 \\ \hline 42 \end{array}$$



## 1.1 Whole Numbers: Using an Area Model to Explain Multiplication

### EXPLORE 2

Pairs

➤ SP4\*-5  
Multiplication Using  
an Area Model

➤ SP6\*  
Area Problems

Math Background 3

Teaching Tip 1

- Students use area models and traditional algorithm to multiply two-digit and three-digit numbers.

### SUMMARIZE 2

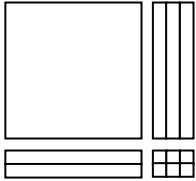
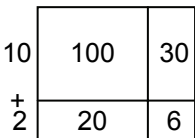
Whole Class

➤ SP6\*-7  
Area Problems

Math Background 3

Teaching Tip 3

- Ask students to record their methods for finding  $12 \times 13$  on the board. Encourage students to share routine and non-routine strategies (for example: 10 groups of 12 is 120, and 3 groups of 12 is 36, so  $120 + 36 = 156$ ).

Rectangle $12 \times 13$	Traditional Algorithm 1	Traditional Algorithm 2
$100 + 20 + 30 + 6 = 156$ 	$\begin{array}{r} 12 \\ 13 \\ \hline 6 \\ 30 \\ 20 \\ \hline 100 \\ 156 \end{array}$ <p><math>6 = 2 \times 3</math> <math>30 = 10 \times 3</math> <math>20 = 10 \times 2</math> <math>100 = 10 \times 10</math></p>	$\begin{array}{r} 12 \\ 13 \\ \hline 36 \\ 120 \\ \hline 156 \end{array}$
 <p>Rectangle not drawn to scale.</p>		

- Use the area model to justify a traditional algorithm.

**Where do the partial products come from?** These are easily identified in the area models and the algorithm.

**Were any of these partial products combined in a traditional algorithm?** Yes, in traditional algorithm #2, only partial products are typically recorded:  $30 + 6 = 36$ ,  $100 + 20 = 120$ .

**In a traditional algorithm we sometimes say, “ $3 \times 2 = 6$  ... write down 6... and then  $3 \times 1 = 3$  ...write down 3.” What does the “ $3 \times 1$ ” represent in the problem?  $3 \times 10$  Where is this partial product in our recordings?**

## 1.1 Whole Numbers: Using an Area Model to Explain Multiplication

### PRACTICE

Individuals

➤ SP6\*-7  
Area Problems  
(continued)

Teaching Tip 3,4

- Students complete SP6-7 for more practice. SP7 is appropriate for homework. Time permitting, invite students to share and discuss solutions. Emphasize the flexibility of the area model and its connection to a traditional algorithm.

***Did anyone get different answers for the same problem when using the area model and a traditional algorithm? What does that mean?***

It is likely that students found their own computational errors when doing problems in two different ways.

***In problem #2,  $18 \times 74$ , the rectangle was divided up in 2 different ways. Did you get the same answer each time? Yes. Why?*** The area of a rectangle is the number of unit squares inside of it. The sum of the areas of the smaller rectangles is the same as the area of the big rectangle.

***In problem #2, which area model did you prefer?*** Some students may prefer the first model because there is less multiplying needed. Some students may prefer the second model because the expanded form uses easier multiplication facts.

- Encourage students to try other expanded forms for 18 and 74 to verify that they will get the same product.

### CLOSURE

Whole Class

➤ SP1\*  
Ready, Set

- Review the goals, standards, and vocabulary for the lesson.

## 1.1 Whole Numbers: Using an Area Model to Explain Multiplication

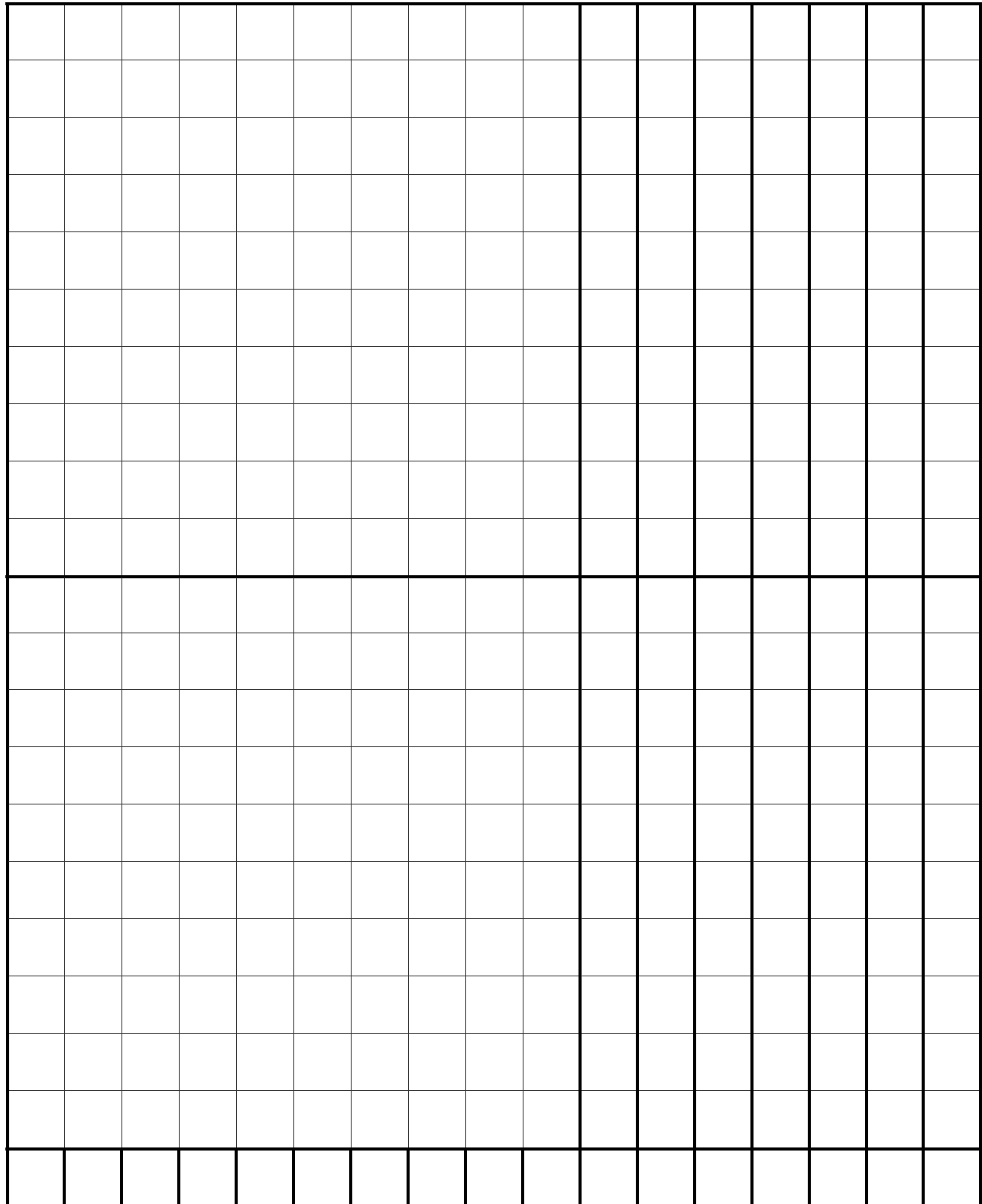
SELECTED SOLUTIONS	
SP1 Warmup	<ol style="list-style-type: none"> <li>1. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20</li> <li>2. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30</li> <li>3. 4, 8, 12, 16, 20, 24, 28, 32, 36, 40</li> <li>4. 5, 10, 15, 20, 25, 30, 35, 40, 45, 50</li> <li>5. 6, 12, 18, 24, 30, 36, 42, 48, 54, 60</li> <li>6. 7, 14, 21, 28, 35, 42, 49, 56, 63, 70</li> <li>7. 8, 16, 24, 32, 40, 48, 56, 64, 72, 80</li> <li>8. 9, 18, 27, 36, 45, 54, 63, 72, 81, 90</li> <li>9. 10, 20, 30, 40, 50, 60, 70, 80, 90, 100</li> </ol>
SP2 Properties of Multiplication	<ol style="list-style-type: none"> <li>1. Associative property of multiplication</li> <li>2. Distributive property</li> <li>3. Commutative property of multiplication</li> <li>4. Answers will vary.</li> <li>5. Answers will vary.</li> <li>6. Answers will vary.</li> </ol>
SP3 Multiplication Made Easy	<ol style="list-style-type: none"> <li>2               <ol style="list-style-type: none"> <li>a. 48</li> <li>b. 480</li> <li>c. 4,800</li> <li>d. 480</li> </ol> </li> <li>3               <ol style="list-style-type: none"> <li>a. 15</li> <li>b. 150,000</li> <li>c. 150</li> <li>d. 150,000</li> </ol> </li> <li>4               <ol style="list-style-type: none"> <li>a. 36</li> <li>b. 3,600</li> <li>c. 36,000</li> <li>d. 3,600,000</li> </ol> </li> <li>5               <ol style="list-style-type: none"> <li>a. 50</li> <li>b. 5,000</li> <li>c. 500</li> <li>d. 5,000</li> </ol> </li> <li>6. Multiply the non-zero digits and then put the appropriate numbers of zeros at the end.</li> </ol>
SP4-5 Multiplication Using an Area Model	<p>In box: Length = 5, width = 3, area = 15 (note that length need not be longer than width)          Multiplication sentence: <math>5 \times 3 = 15</math> or <math>3 \times 5 = 15</math></p> <ol style="list-style-type: none"> <li>1. Commutative property</li> <li>2. <math>7 \times 3 = 21</math>; <math>3(5 + 2) = 6 + 15 = 21</math>; <math>(2 + 1)7 = 14 + 7 = 21</math>; we add the areas of the small rectangles to find the area of the bigger rectangle</li> <li>3. Yes. This shows the distributive property. <math>3(7) = 14 + 7</math></li> <li>4. <math>3 \times 14 = 42</math>; <math>3 \times (10 + 4) = 30 + 12 = 42</math></li> <li>5. The area model partitions the rectangle into smaller rectangles that correspond to the partial products in the algorithm.</li> <li>6. Distributive property</li> <li>7. Associative property of multiplication (after renaming 50 as <math>5 \times 10</math>)</li> <li>8. Commutative property of multiplication</li> </ol>
SP6-7 Area Problems	<ol style="list-style-type: none"> <li>1. 156</li> <li>2. 1,332</li> <li>3. 2,970</li> <li>4. 57,190</li> </ol>



# **REPRODUCIBLES**

- 0.3 Whole Numbers: Vocabulary, Addition and Subtraction
- 1.1 Whole Numbers: Using an Area Model to Explain Multiplication
- 2.2 Whole Numbers: Division with Remainder

## **BASE-10 BLOCKS**



**STUDENT PAGES**

## WHOLE NUMBERS: USING AN AREA MODEL TO EXPLAIN MULTIPLICATION

Ready (Summary)	Set (Goals)
<p>We will learn strategies for recalling multiplication facts. We will use an area model to multiply numbers. We will learn and use properties of multiplication.</p>	<ul style="list-style-type: none"><li>• Use logical reasoning to derive multiplication facts.</li><li>• Use associative and commutative properties of multiplication.</li><li>• Use the distributive property.</li><li>• Use expanded notation.</li><li>• Compare the traditional algorithm for multiplication to an area model for multiplication.</li></ul>

### Go (Warmup)

Write the next seven numbers in each skip counting pattern:

1. 2 4 6 \_\_\_\_\_
2. 3 6 9 \_\_\_\_\_
3. 4 8 12 \_\_\_\_\_
4. 5 10 15 \_\_\_\_\_
5. 6 12 18 \_\_\_\_\_
6. 7 14 21 \_\_\_\_\_
7. 8 16 24 \_\_\_\_\_
8. 9 18 27 \_\_\_\_\_
9. 10 20 30 \_\_\_\_\_

## PROPERTIES OF MULTIPLICATION

Associative property of multiplication: The identity  $(ab)c = a(bc)$  holds for all numbers  $a$ ,  $b$ , and  $c$ . In other words, the product does not depend on the grouping of the factors. We can multiply  $a$  and  $b$  first, and then multiply the product by  $c$ ; or we can multiply  $b$  and  $c$  first, and then multiply the product by  $a$ .

Example:  $(3 \cdot 4)(5) = (12)(5) = 60$  and  $(3)(4 \cdot 5) = (3)(20) = 60$

Commutative property of multiplication: The identity  $ab = ba$  holds for all numbers  $a$  and  $b$ . In other words, the product does not depend on the order of the factors.

Example:  $3 \cdot 5 = 5 \cdot 3 = 15$

Distributive property: The identities  $a(b + c) = ab + ac$  and  $(b + c)a = ba + ca$  hold for all numbers  $a$ ,  $b$ , and  $c$ . This property relates two operations (multiplication and addition). It is called the “distributive property” because it “distributes” the factor outside the parentheses over the two terms within the parentheses.

Example:  $3(4 + 5) = 3(9) = 27$  and  $3(4) + 3(5) = 12 + 15 = 27$

For #1-3, write the property illustrated by each equation.

1.  $(2 \times 5) \times 4 = 2 \times (5 \times 4)$  \_\_\_\_\_

2.  $2(7 + 4) = (2 \times 7) + (2 \times 4)$  \_\_\_\_\_

3.  $(4)(2) = (2)(4)$  \_\_\_\_\_

For #4-6, write a number sentence that illustrates each property.

4. Commutative property of multiplication \_\_\_\_\_

5. Associative property of multiplication \_\_\_\_\_

6. Distributive property \_\_\_\_\_

## MULTIPLICATION MADE EASY

### Derived facts

$8 \times 7 = 56$

$6 \times 9 = 54$

$7 \times 9 = 63$

$3 \times 7 = 21$

$7 \times 6 = 42$

$6 \times 8 = 48$

1. Many students find these multiplication facts difficult to remember. Choose one of these facts and explain how to derive it in two different ways.

### Multiplying by Multiples of 10

Find each product.

2.

a.  $6 \times 8$  \_\_\_\_\_

b.  $60 \times 8$  \_\_\_\_\_

c.  $60 \times 80$  \_\_\_\_\_

d.  $80 \times 6$  \_\_\_\_\_

3.

a.  $5 \times 3$  \_\_\_\_\_

b.  $500 \times 300$  \_\_\_\_\_

c.  $3 \times 50$  \_\_\_\_\_

d.  $3,000 \times 50$  \_\_\_\_\_

4.

a.  $9 \times 4$  \_\_\_\_\_

b.  $90 \times 40$  \_\_\_\_\_

c.  $400 \times 90$  \_\_\_\_\_

d.  $4,000 \times 900$  \_\_\_\_\_

5.

a.  $10 \times 5$  \_\_\_\_\_

b.  $100 \times 50$  \_\_\_\_\_

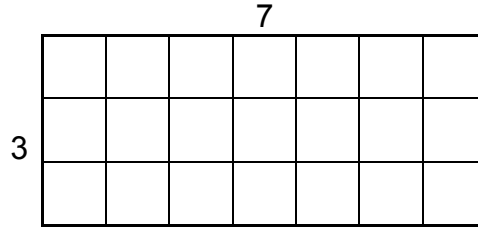
c.  $10 \times 50$  \_\_\_\_\_

d.  $50 \times 100$  \_\_\_\_\_

6. Describe a strategy for easily multiplying by a multiple of 10.

## MULTIPLICATION USING AN AREA MODEL

Area is the number of square units inside a figure. To find the area of a rectangle, multiply the length by the width.



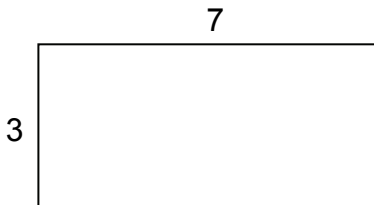
length = \_\_\_\_\_ width = \_\_\_\_\_ area = \_\_\_\_\_

Multiplication sentence: \_\_\_\_\_  $\times$  \_\_\_\_\_ = \_\_\_\_\_ or

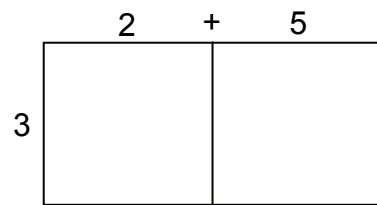
\_\_\_\_\_  $\times$  \_\_\_\_\_ = \_\_\_\_\_

1. What property is illustrated by  $3 \times 7 = 7 \times 3$ ? \_\_\_\_\_

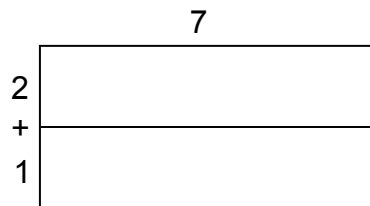
2. Here are three rectangles. Write multiplication sentences suggested by the rectangles.



\_\_\_\_\_  $\times$  \_\_\_\_\_ = \_\_\_\_\_



$3 \times (\text{_____} + \text{_____}) = \text{_____} + \text{_____} = \text{_____}$



$(\text{_____} + \text{_____}) \times \text{_____} = \text{_____} + \text{_____} = \text{_____}$

*Rectangles are not drawn to scale.*

1.1 Whole Numbers: Using an Area Model to Explain Multiplication

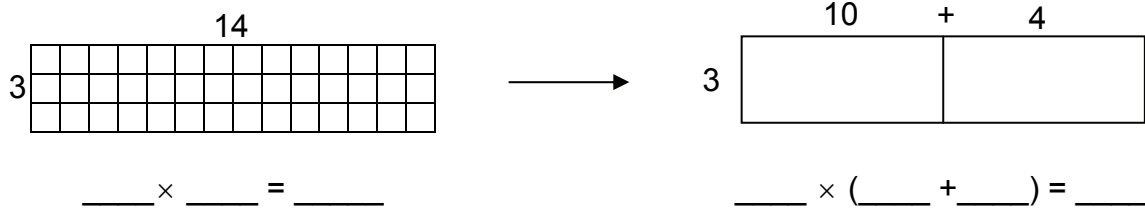
## MULTIPLICATION USING AN AREA MODEL (continued)

3. Does  $(2 + 1)7 = (2)(7) + (1)(7)$ ? Explain.

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4. Multiply  $3 \times 14$  using an area model strategy.



*Rectangles are not drawn to scale.*

5. Multiply  $3 \times 14$  using a traditional algorithm. How do the steps of the algorithm connect to the area model?

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State the property illustrated by each example.

6.  $3 \cdot (10 + 4) = 3 \cdot 10 + 3 \cdot 4$  \_\_\_\_\_

7.  $3 \times 50 = 3 \times (5 \times 10) = (3 \times 5) \times 10$  \_\_\_\_\_

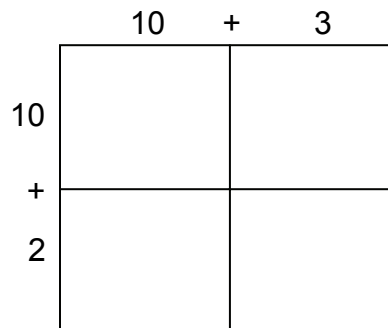
8.  $3(10 + 4) = 3(4 + 10)$  \_\_\_\_\_

1.1 Whole Numbers: Using an Area Model to Explain Multiplication

## AREA PROBLEMS

1. Multiply using a traditional algorithm and an area model.

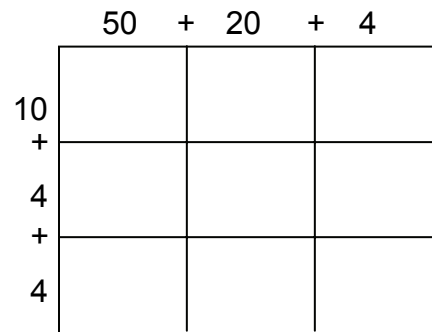
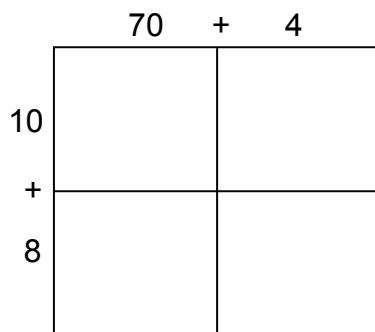
$$\begin{array}{r} 12 \\ \times 13 \\ \hline \end{array}$$



Draw arrows to show how the partial products of a traditional algorithm and rectangles inside the rectangle of an area model are related.

2. Compute  $18 \times 74$  using a traditional algorithm. Multiply using an area model in two different ways.

$$\begin{array}{r} 18 \\ \times 74 \\ \hline \end{array}$$



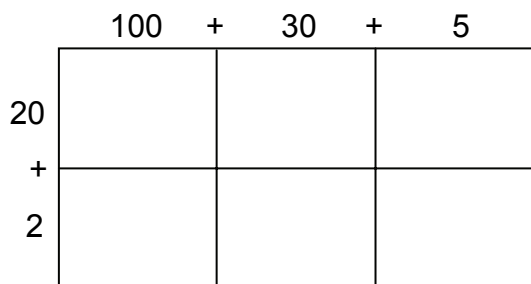
Draw arrows to show how the partial products of the traditional algorithm and rectangles inside the rectangle of the area models are related.

*Rectangles are not drawn to scale.*

1.1 Whole Numbers: Using an Area Model to Explain Multiplication

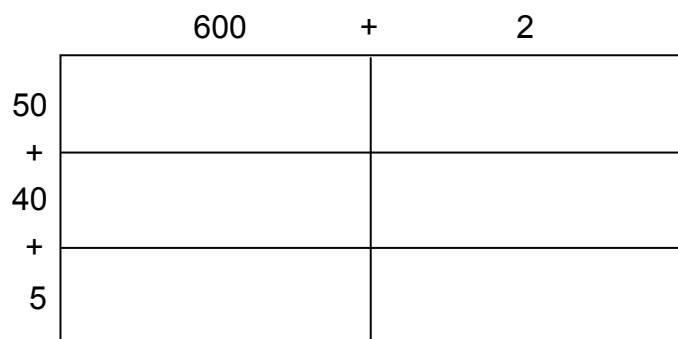
## AREA PROBLEMS (continued)

3. Use an area model to compute  $135 \times 22$ . Check your answer using a traditional algorithm.



Check:

4. Use an area model to compute  $602 \times 95$ . Check your answer using a traditional algorithm.



Check:

5. Make up a challenging multiplication problem. Use an area model to multiply the numbers. Then check your answer using a traditional algorithm.

*Rectangles are not drawn to scale.*