USING ALGEBRA TO PROVE CONJECTURES 1

In this lesson, students write algebraic expressions that represent verbal descriptions. Students simplify algebraic expressions as they perform number tricks and show how the tricks work.

This lesson falls near the middle of a lesson cluster that focuses on symbolic manipulation of expressions and equations. In previous lessons, students distinguished between expressions and equations, practiced writing and evaluating simple expressions, learned the importance of maintaining balance in an equation, and developed strategies for solving simple equations. In later lessons, students will prove conjectures using algebraic expressions and evaluate more complicated expressions using the conventions for order of operations. They will also learn how to manipulate and solve more complex linear equations.

Math Goals (Standards for posting in **bold**)	• Use algebraic expressions to generalize patterns. (**Gr7 AF1.3**; Gr7 MR1.1; **Gr7 MR2.4**; **Gr7 MR3.0**; Gr7 MR3.3) • Apply number properties to simplify algebraic expressions. (Gr7 AF1.3) • Evaluate expressions using rational numbers. (Gr4 AF1.2; Gr4 AF1.3; Gr7 NS1.2) • Write verbal expressions as algebraic expressions. (Gr6 AF1.0; Gr7 AF1.1; ALG4.0)
Summative Assessment	Future Weeks • Week 28: Writing Algebraic Expressions (Gr7 AF1.0; Gr7 AF1.1) • Week 29: Simplifying Expressions, Properties of Arithmetic (Gr7 AF1.3)

PLANNING INFORMATION

Estimated Time: 30 – 45 Minutes

Student Pages	**Materials**	**Reproducibles**
* SP1: Ready, Set, Go * SP2: Number Trick 1 * SP3: Number Trick 2 SP4: Number Trick 3 SP5: Number Trick Template	Calculators (optional) Cups and markers Resealable plastic bags	* R95: Number Trick Template
Homework	**Prepare Ahead**	**Management Reminders**
SP4: More Number Tricks SP5: Number Trick Template	Be sure to draw some of the tricks ahead of time to get a sense of the pictorial process.	Provide cups and markers in baggies to students who need them.
Assessment	**Strategies for English Learners**	**Strategies for Special Learners**
* SP20-21: Knowledge Check 25 R97-98: Knowledge Challenge 25 A125: Weekly Quiz 25	Monitor students' ability to translate verbal expressions given orally, and in writing. Preview vocabulary used in algebraic expressions.	Use concrete manipulatives, or a model of cups and markers, to demonstrate the steps in the number tricks as students write the algebraic expressions.

* Recommended transparency: Blackline masters for overheads 293-296 and 302-303 can be found in the Teacher Resource Binder.

THE WORD BANK	
expression	A mathematical expression is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number. Example: Some mathematical expressions are $7x$, $a + b$, $4v - w$, and 19.
term	A term in a mathematical expression involving addition (or subtraction) is a quantity being added (or subtracted). Terms that are (nonzero) constant multiples of each other are referred to as like terms. Example: In the expression $x^2 + 2x + y - 3x + 6x^2$, there are five terms. The terms $2x$ and $3x$ are like terms, since $3x$ is $\frac{3}{2}$ times $2x$. The terms x^2 and $6x^2$ are also like terms.
generalization	Generalization is the process of formulating general concepts by abstracting common properties from specific cases. A generalization of a theorem is a statement that includes the theorem as a special case. Example: The associative, commutative, and distributive properties are properties of numbers that have been generalized to apply to more general algebraic systems.
conjecture	A conjecture is a statement that is proposed to be true, but has not been proven to be true nor to be false. Example: The twin prime conjecture asserts that there are infinitely many prime numbers p such that $p + 2$ is also prime. There are many examples of twin primes, such as 3 and 5, 29 and 31, 101 and 103, but it is not known whether there are infinitely many such pairs of primes.
proof	A proof of a mathematical statement is an argument based on definitions, previously established theorems, and accepted rules of logic, to justify the statement.

MATH BACKGROUND

Conjecture vs. Proof

Math Background 1

Preview/Warmup

Here is an example of a "number trick." ***Choose a number, multiply by 4, add 6, subtract the original number, divide by 3, and subtract 2. What number do you have now?*** Answer: The original number.

A number trick	Using specific cases		Using a variable
Choose a number	3	0.5	n
Multiply by 4	12	2	$4n$
Add 6	18	8	$4n + 6$
Subtract the original number	15	7.5	$3n + 6$
Divide by 3	5	2.5	$n + 2$
Subtract 2	3	0.5	n

Based on the results with specific numbers, we conjecture that the result is the original number.

By following the steps of this trick using n to represent any number, we prove that the result is the original number for all numbers.

When students in a classroom select different numbers to test a number trick such as this one, they may convince themselves that the trick will always work. Their generalization is a conjecture because it has not been proven to be true nor shown to be false. The sort of question, "Will this trick work for **all** numbers?" is a very important one in mathematics. Certainly it is impossible to try **all** numbers. The use of symbolic algebra for the purpose of generalization is an efficient way to prove this conjecture and it provides a convincing way to show the usefulness of algebra.

Manipulating Expressions

Math Background 2

Introduce

Some common procedures for manipulating expressions:

To evaluate an expression, replace each variable in the expression with a number and then calculate the value of the expression.

Example: If $x = 2$, then $6 + 3x = 6 + 3(2) = 12$. Here the expression $6 + 3x$ has been evaluated at $x = 2$.

To combine like terms, rewrite the sum or difference of like terms as a single term.

Example: $3x + 4x = (3 + 4)x = 7x$

To simplify an expression, remove all parentheses (using the distributive property if necessary) and combine like terms.

Example: $4(2x + 3) + x = 8x + 12 + x = 9x + 12$

PREVIEW / WARMUP

Whole Class

- SP1* Ready, Set, Go

Math Background 1

Calculators

- Introduce the goals and standards for the lesson. Discuss important vocabulary as relevant.
- Students use arithmetic to test a number trick.

  With what number did you end? Students should say 4.

  Will this trick work for all numbers? Responses may vary. This will be proven later.

  Why do you think that it works? Answers will vary. Leave the question open for later discussion.

INTRODUCE

Whole Class

- SP2* Number Trick 1

Math Background 2

Cups and markers
Resealable plastic bags
Calculators

- Lead students through the process of choosing a small natural number and performing the required arithmetic.

  With what number did you end? Students should respond that their starting and ending numbers are the same.

- Ask for a show of hands to verify that starting and ending numbers are the same. For those whose results are different, challenge the class to work through the steps for these numbers to convince students that the result is the original number.

  Does this trick work for all numbers? Responses may vary. This will be proven later.

  Are there some numbers for which this trick will not work? Again, responses may vary. Challenge students to test the trick with small negative integers, simple fractions, or decimals. At this point, students may think that they have done enough examples to convince themselves that it does work for all numbers.

  Did we try every number? No. ***Can we?*** No. We would have to try infinitely many numbers, which is impossible. ***How can we prove that this trick works for all numbers?*** We cannot try all numbers, but we can generalize using pictures and/or algebra.

- Walk students through translating the words into boxes (for variables) and dots (for numbers) to prove that the number trick works for all numbers. Then transition to a variable such as *n*. Use a cups and markers model if desired.

EXPLORE

Individuals/Pairs

- SP3* Number Trick 2

Cups and markers
Resealable plastic bags
Calculators

- Students perform another number trick and check their results with each other. After they convince themselves by experimenting with numbers, students prove their conjecture pictorially and algebraically. Share at the overhead as desired.

SUMMARIZE

Whole Class

- SP3* Number Trick 2

- Discuss the number trick and the proof on SP3, verifying that the trick always works.

  What is the number trick? The ending number is twice the starting number.

  How many examples would you have to try to prove that it always works? Infinitely many, which is impossible.

- Select students to share the process for this number trick and for the warmup at the overhead.

PRACTICE

Individuals/Pairs

- SP4 Number Trick 3

Cups and markers
Resealable plastic bags
Calculators

- Students complete more number tricks and prove that they work for all numbers. For more computation practice, ask students to test an integer, fraction, or a decimal. This is appropriate for homework.

EXTEND

Individual/Small Groups

- SP5, R95* Number Trick Template

- Challenge students to make up their own simple number trick. Suggest a reasonable number of steps, say 10 or less. Encourage students to follow a similar format to the one practiced in class.

- Invite some students to write the number tricks on an overhead or read the steps of their tricks to the class. Ask the class to try the trick with simple numbers, then verify whether the trick works for all numbers.

CLOSURE

Whole Class

- SP1* Ready, Set

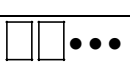
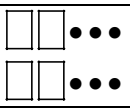
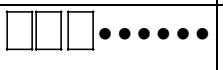
- Review the goals, standards, and vocabulary for the lesson.

SELECTED SOLUTIONS

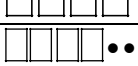
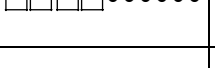
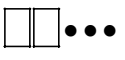
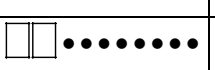
SP1 Warmup

The resulting number is 4.

SP2 Number Trick 1

Steps	Words	Number	Pictures	Algebraic Process
1	Choose a single- digit, natural number.	5		n
2	Add the number to itself.	5 + 5 =10		$2n$
3	Add 3.	10 + 3 = 13		$2n + 3$
4	Double the result.	2(13) = 26		$2(2n + 3) = 4n + 6$
5	Subtract your original number.	26 – 5 = 21		$3n + 6$
6	Divide by 3.	$21 \div 3 = 7$		$n + 2$
7	Subtract 2.	7 – 2 = 5		n
8	What number do you have now?	5 (which is the original number)		n

SP3 Number Trick 2

Steps	Words	Number	Pictures	Algebraic Process
1	Choose a natural number.	3		n
2	Multiply by 4.	4(3) = 12		$4n$
3	Add 6.	12 + 6 = 18		$4n + 6$
4	Multiply by $\frac{1}{2}$.	$\frac{1}{2}(18) = 9$		$\frac{1}{2}(4n + 6)$ $= 2n + 3$
5	Add 5.	9 + 5 = 14		$2n + 3 + 5$ $= 2n + 8$
6	Divide by 2.	$14 \div 2 = 7$		$(2n + 8) \div 2$ $= n + 4$
7	Subtract 4.	7 – 4 = 3		$n + 4 – 4$ $= n$
8	Add your original number.	3 + 3 = 6		$n + n = 2n$
9	What number do you have now?	6 (which is twice the original number)		$2n$

	Steps	Words	Number		Pictures	Algebraic Process
	1	Choose a number.	0.75		□	n
	2	Add 4.	0.75 + 4 = 4.75		□ ••••	$n + 4$
	3	Multiply by 2.	4.75(2) = 9.5		□ •••• □ ••••	$2(n + 4) = 2n + 8$
	4	Subtract 8.	9.5 – 8 = 1.5		□ □	$2n + 8 – 8 = 2n$
	5	Divide by 2.	$\frac{1.5}{2}$ = 0.75		□	$2n \div 2 = n$
	6	What is the result?	0.75 (which is the original number)		□	n
SP4 Number Trick 3	**Steps**	**Words**	**Number**		**Pictures**	**Algebraic Process**
	1	Choose a number.	-0.5	$\frac{3}{5}$	□	n
	2	Add 3.	2.5	$\frac{18}{5}$	□ •••	$n + 3$
	3	Multiply by 2.	5	$\frac{36}{5}$	□ ••• □ •••	$2(n + 3) = 2n + 6$
	4	Subtract 4.	1	$\frac{16}{5}$	□ □ ••	$2n + 6 – 4 = 2n + 2$
	5	Multiply by $\frac{1}{2}$.	0.5	$\frac{16}{10}$	□ •	$\frac{1}{2}(2n + 2) = n + 1$
	6	Subtract the original number.	1	$\frac{10}{10}$	•	$n + 1 – n$
	7	What is the result?	1	1	•	1

REPRODUCIBLES

NUMBER TRICK TEMPLATE

Use this page to create your own number trick.

Step	Words	Numbers	Pictures	Algebraic Process

What is the number trick? _____

_____

Does this trick always work? Explain. _____

_____

STUDENT PAGES

USING ALGEBRA TO PROVE CONJECTURES 1

Ready (Summary)	Set (Goals)
We will perform mathematical number tricks and use algebraic expressions to show how they work.	• Use algebraic expressions to generalize patterns. • Apply number properties to simplify algebraic expressions. • Evaluate expressions using rational numbers. • Write verbal expressions as algebraic expressions.

Go (Warmup)

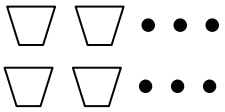
Perform the number trick below.

Steps	Directions	Result
1	Choose a natural number between 1 and 10.	
2	Multiply your number by 2.	
3	Add 8 to your answer.	
4	Divide your answer by 2.	
5	Subtract your original number from your answer.	

What number did you end with? Compare your answers with other classmates' answers. Will this trick work for all numbers? Explain.

NUMBER TRICK 1

Perform the number trick below.

Steps	Words	Numbers	Pictures	Algebraic Process
1	Choose a single digit natural number.			n
2	Add the number to itself.			$n + n = 2n$
3	Add 3.			$2n + 3$
4	Double the result.			$2(2n + 3) = 4n + 6$
5	Subtract your original number.			
6	Divide by 3.			
7	Subtract 2.			
8	What number do you have now?			

What is the number trick? _____

_____

Does this trick always work? Explain. _____

_____

NUMBER TRICK 2

Perform the number trick below.

Steps	Words	Numbers	Pictures	Algebraic Process
1	Choose a natural number.			
2	Multiply by 4.			
3	Add 6.			
4	Multiply by $\frac{1}{2}$.			
5	Add 5.			
6	Divide by 2.			
7	Subtract 4.			
8	Add your original number.			
9	What number do you have now?			

What is the number trick? _____

_____

Does this trick always work? Explain. _____

_____

NUMBER TRICK 3

Perform the number trick below.

Steps	Words	Numbers	Pictures	Algebraic Process
1	Choose a number.			n
2	Add 4.			$n + 4$
3	Multiply by 2.			
4	Subtract 8.			
5	Divide by 2.			
6	What is the result?			

What is the number trick? _____

Does this always work? Explain. _____

Steps	Words	Numbers	Pictures	Algebraic Process
1	Choose a number.			n
2	Add 3.			
3	Multiply by 2.			
4	Subtract 4.			
5	Multiply by $\frac{1}{2}$.			
6	Subtract the original number.			
7	What is the result?			

What is the number trick? _____

Does this trick always work? Explain. _____

NUMBER TRICK TEMPLATE

Use this page to create your own number trick.

Steps	Words	Numbers	Pictures	Algebraic Process

What is the number trick? _____

_____

Does this trick always work? Explain. _____

_____