

RECTANGLES: FIXED AREA

Students review the formulas for the area and perimeter of rectangles. They create different rectangles with a fixed area, organize the information in tables, and graph various relationships.

This lesson falls at the beginning of a lesson cluster that applies algebra readiness concepts to length, area, and volume. In earlier lessons, students explored length and area for rectangles and circles, and they explored surface area and volume for prisms and cylinders. At the end of the cluster, students will use “cut-up proofs” to derive formulas for other polygons. Algebra connections are emphasized in all of the geometry lessons through the solution of problems using geometry formulas and through the graphing of functional relationships.

Math Goals

(Standards for posting in **bold**)

- Find lengths of horizontal and vertical line segments on a coordinate system.
(Gr4 MG2.2; Gr4 MG2.3; Gr7 MG3.2)
- Identify and graph coordinates in four quadrants of the coordinate plane.
(Gr4 MG2.0; Gr5 AF1.4)
- Find the perimeters of rectangles with fixed areas.
(Gr4 MG1.0; Gr6 AF3.1)
- Use formulas for the perimeter and area of rectangles to solve problems.
(Gr4 MG1.0; Gr7 MG2.1)
- Interpret the meaning of graphs.
(Gr5 AF1.0; **Gr7 AF1.5**; Gr7 MR1.2)

Summative Assessment

Future Week

- **Week 24: Finding Lengths and Areas**
(Gr4 MG2.2; Gr4 MG2.3; Gr5 AF1.4; Gr7 MG2.1; Gr7 MG3.2)

13.1 Rectangles: Fixed Area

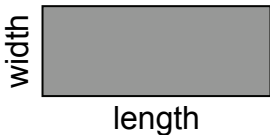
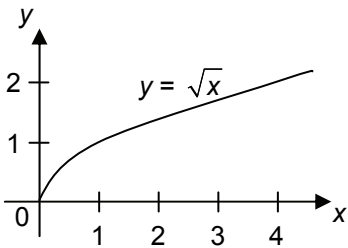
PLANNING INFORMATION

Estimated Time: 45 – 60 Minutes

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Student Pages * SP1: Ready, Set, Go * SP2: Area = 12 units ² * SP3: Fixed Area Rectangles * SP4: Fixed Area Tables * SP5: Fixed Area Graph 1: Length vs. Width * SP6: Fixed Area Graph 2: Length vs. Perimeter * SP7: Fixed Area Graph 3: Length vs. Area * SP8: Graphing Rectangles	Materials Square tiles	Reproducibles
Homework * SP8: Graphing Rectangles	Prepare Ahead	Management Reminders
Assessment * SP21: Knowledge Check 13 R55-56: Knowledge Challenge 13 A57-58: Weekly Quiz 13	Strategies for English Learners Maintain routines that build vocabulary such as the math word wall and vocabulary cards.	Strategies for Special Learners Use models to help students understand the problem To help students understand the meaning of perimeter, emphasize that <u>perimeter</u> is the <u>rim</u> of the figure.

* Recommended transparency: Blackline masters for overheads 139-146 and 152 can be found in the Teacher Resource Binder.

13.1 Rectangles: Fixed Area

THE WORD BANK	
length	The <u>length</u> of a curve is a measure of distance along the curve.
area	<p>The <u>area</u> of a two-dimensional figure is a measure of the size of the figure, expressed in square units. The <u>area of a rectangle</u> is the product of its length and its width. The area of the union of non-overlapping figures is the sum of their areas.</p> <div style="text-align: center;">  <p style="margin-left: 100px;">Area = (width) • (length)</p> </div> <p>Example: If a rectangle has a length of 12 inches and a width of 5 inches, its area is $(5)(12) = 60$ square inches.</p>
perimeter	<p>The <u>perimeter</u> of a plane figure is the length of the boundary of the figure.</p> <p>Example: The perimeter of a square is four times its side-length. The perimeter of a rectangle is twice the length plus twice the width. The perimeter of a disk is π times its diameter.</p>
function	<p>A <u>function</u> f on a domain D is a rule that assigns to each element x of D a unique value $y = f(x)$. (Read $f(x)$ as “f of x” or “the value of f at x”, not “f times x”.) Thus a function is an input-output rule that assigns to each input x a unique output $y = f(x)$.</p> <p>Example: The function $y = mx + b$ assigns to each real number x the value $y = f(x) = mx + b$.</p>
graph of a function	<p>The <u>graph of a function</u> f on a domain D is the collection of points with coordinates (x, y), where x is in D and $y = f(x)$ is the value of the function at x.</p> <p>Example: The graph of the function $y = mx + b$ is the straight line in the plane consisting of the pairs $(x, mx + b)$. The graph of $f(x) = \sqrt{x}$, $x \geq 0$, is depicted below.</p> <div style="text-align: center;">  </div>

13.1 Rectangles: Fixed Area

MATH BACKGROUND

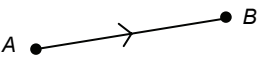

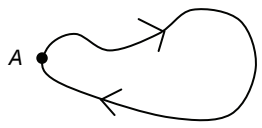
Vocabulary Alert: What is the Area of a Polygon?

<p>Math Background 1 Introduce</p>	<p>What is the area of a line segment? Since a line segment can be enclosed by rectangles of arbitrarily small area, a line segment has area zero. What is the area of a polygon? Since a polygon consists of finitely many line segments, does a polygon have area zero? Strictly speaking, yes! However, when we refer to the <u>area of a polygon</u>, we mean the area of the polygonal region enclosed by the polygon.</p> <p>A figure in a plane whose boundary is a polygon is called a <u>polygonal figure</u>. We use the word <u>polygon</u> also to refer to a polygonal figure. The meaning of polygon must be inferred from context. Thus the area of a triangle refers to the area of the triangular figure enclosed by the three sides of the triangle.</p> <p>In mathematics, it is not uncommon to use vocabulary with ambiguous meanings, whose interpretation must be inferred from context. We often use the word “number” without specifying whether the number is a natural number, whole number, integer, real number, or even complex number. We use such words as “fraction” and “decimal” without specifying whether we mean the representation or the number represented.</p>
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Which Side of a Rectangle is its Base?

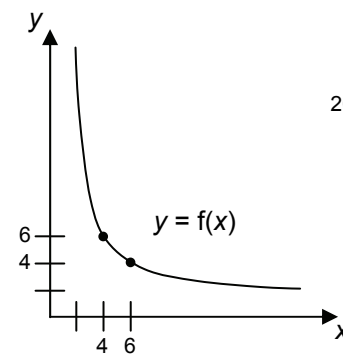
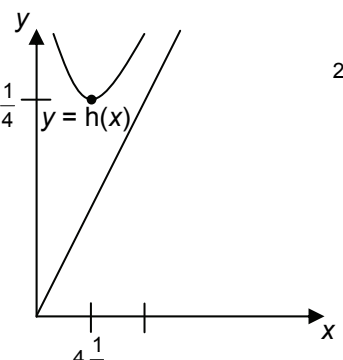
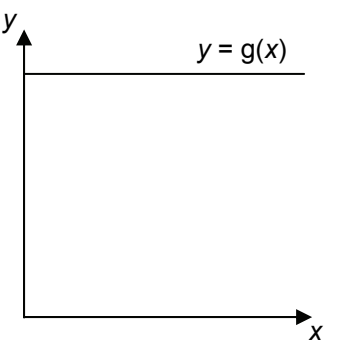
<p>Math Background 2 Introduce</p>	<p>We may select any side of the rectangle and refer to it as the <u>base</u> of the rectangle. In the formula,</p> $\text{Area} = (\text{length}) \bullet (\text{width}),$ <p>the “length” then refers to the length of the base, while the “width” refers to the length of the sides perpendicular to the base. The width is then the same as the height of the rectangle, and the formula for the area can also be written</p> $\text{Area} = (\text{length of base}) \bullet (\text{height}).$ <p>Similarly, we may select any side of a triangle and declare it to be the base of the triangle.</p>
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Lengths of Curves

<p>Math Background 3 Teacher Mathematical Insight</p>	<p>A <u>polygonal path</u> is a linking of straight line paths end-to-end. A <u>closed path</u> is a path that starts and ends at the same point.</p> <div style="display: flex; justify-content: space-around; align-items: flex-end;"> <div style="text-align: center;">  <p>straight line path</p> </div> <div style="text-align: center;">  <p>polygonal path</p> </div> <div style="text-align: center;">  <p>simple closed path</p> </div> </div> <p>Once a unit length is specified, the length of any straight line segment can be determined. The <u>length of a polygonal path</u> is found by adding the lengths of the line segments making up the path. The length of a more complicated curve is defined to be the limit of the lengths of polygonal paths that approximate the curve.</p>
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13.1 Rectangles: Fixed Area

MATH BACKGROUND (continued)

Fixed Area Graphs			
	Dependent Variable (output)	Explicit Rule	Description of Graph
Math Background 4 Teacher Mathematical Insight	width	$f(x) = \frac{24}{x}$	hyperbola, the axes $x = 0$ and $y = 0$ are asymptotes
	perimeter	$h(x) = 2\left(x + \frac{24}{x}\right)$ or $h(x) = \frac{2x^2 + 48}{x}$	hyperbola, the lines $y = 2x$ and $x = 0$ are asymptotes $h(x)$ attains minimum value of $20\frac{1}{4}$ at $x = 4\frac{1}{2}$, corresponding to low point $\left(4\frac{1}{2}, 20\frac{1}{4}\right)$ on graph.
	area	$g(x) = 24$	constant function, graph is a horizontal line
			

13.1 Rectangles: Fixed Area

TEACHING TIPS

Student Misconception: Computing Length Around Corners Inaccurately

<p>Teaching Tip 1</p> <p>Introduce</p>	<p>Perimeter is a length. A concrete way to find the perimeter of a rectangle is to count the unit lengths around the <u>rim</u> of the rectangle. However, watch for this student counting error:</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr><td>10</td><td></td><td></td><td>5</td></tr> <tr><td>9</td><td>8</td><td>7</td><td>6</td></tr> </table> <table style="border-collapse: collapse; text-align: center;"> <tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td></td></tr> <tr><td>14</td><td></td><td></td><td></td><td></td><td>5</td></tr> <tr><td>13</td><td></td><td></td><td></td><td></td><td>6</td></tr> <tr><td>12</td><td></td><td></td><td></td><td></td><td>7</td></tr> <tr><td></td><td>11</td><td>10</td><td>9</td><td>8</td><td></td></tr> </table> </div>	1	2	3	4	10			5	9	8	7	6		1	2	3	4		14					5	13					6	12					7		11	10	9	8	
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Student Misconception: Confusing Perimeter and Area

<p>Teaching Tip 2</p> <p>Teacher Mathematical Insight</p>	<p>Students often confuse perimeter and area, and consequently teachers spend a fair amount of class time treating the concepts together. Mathematically the issue is clear. <u>A perimeter is a length, which is a one-dimensional concept.</u> <u>Area is a two-dimensional concept.</u> One strategy for separating the notions of perimeter and length is to place more emphasis on the concept of length, measuring lengths of various curves and finding distances between points along various paths, before treating the very special case of measuring lengths of closed curves surrounding regions.</p>
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13.1 Rectangles: Fixed Area

PREVIEW / WARMUP

Whole Class

➤ SP1*
Ready, Set, Go

- Introduce the goals and standards for the lesson. Discuss important vocabulary as relevant.
- Students identify the coordinates of each vertex. Then, they find the perimeter and area of rectangle $QRST$. Review how to locate coordinates in four quadrants and how to find perimeter and area if needed.

INTRODUCE

Whole Class

➤ SP2*
Area = 12 units²

Math Background 1, 2

Teaching Tip 1

- Use 12 square tiles to build a 3x4 rectangle at the overhead.

What is the area of this rectangle? 12 square units. **How do you know?** Counting, recognizing 3 rows of 4 squares, or using the area formula $A = LW$.

What is the perimeter of this rectangle? 14 linear units. **How do you know?** Counting, recognizing that there are two lengths of 3 and two lengths of 4, and using any number of methods for finding the sum, or using a perimeter formula, such as $P = 2L + 2W$ or $P = 2(L + W)$.

What other rectangles can be built with a fixed area of 12 square units using whole numbers? A 2 by 6 and a 1 by 12 rectangle.

- Demonstrate how students should draw rectangles on grid paper and record the data. Remind them to record each rectangle along with its rotated counterpart. Encourage students to record the dimensions for the rectangles with the length in ascending order so that they can see that as length increases, width decreases.

How are the dimensions of the rectangle related to its area? The lengths are the factors of the area.

13.1 Rectangles: Fixed Area

EXPLORE

Pairs/ Individuals

- SP3*
Fixed Area
Rectangles
- SP4*
Fixed Area Tables

- Students use 24 square tiles to create rectangles with whole number dimensions and draw them on grid paper. They record the dimensions of each rectangle in the table. Again, encourage students to record the dimensions for the rectangles with the length in ascending order so they can observe the inverse relationship between length and width.

SUMMARIZE

Whole Class

- SP3*
Fixed Area
Rectangles
- SP4*
Fixed Area Tables

- Discuss the results.

What pattern(s) do you notice as you look at the length and width for each rectangle? As the length increases, the width decreases, so that the product of the length and width remain constant. We say that the length is inversely proportional to the width.

What operation(s) can be performed on the length and width values to arrive at different area values? length \times width = area, where area = 24 sq. units.

Which rectangles have the greatest perimeter? The 1 by 24 and 24 by 1 rectangles. ***How do these rectangles look?*** Long and narrow.

Which rectangles have the least perimeter? The 4 by 6 or 6 by 4 rectangles. ***How do these rectangles look?*** Close to squares.

What conjectures can you make about perimeters of rectangles with fixed areas? Assuming that the area is fixed, long and narrow rectangles have the greatest perimeter, while rectangles that look like squares or close to squares have the least perimeter. Allow students exploration time for the opportunity of discovering this themselves. Students may come up with other conjectures as well. Encourage them to test these out later.

(Extension) ***Is it possible to make a rectangle that has an area of 24 square units and a perimeter less than 20 units?*** Yes. For example, if $L = 5$ and $W = 4\frac{4}{5}$, then $A = 24$ and $P = 19\frac{3}{5}$.

13.1 Rectangles: Fixed Area

EXTEND

Whole Class

- SP5-7*
Fixed Area Graphs
1-3

- Challenge students to make fixed area graphs of length vs. width, length vs. perimeter, and length vs. area. Before students begin, discuss appropriate scaling of axes.

What is an appropriate scale for the horizontal axis for Graph #1?

By ones. **the vertical axis?** By ones. **What is an appropriate title for this graph?** Changes in width; width vs. length, etc.

- After students complete their graphs, discuss their features.

(Graph #1) **When the points are connected and the curve is extended, do you think the curve will ever cross the y-axis?** No.

What do you think this means? The product of the length and width is 24, so as the length gets close to zero, the width gets infinitely long.

(Graph #2) **What are the coordinates of the point that has the smallest y-value? What does this point represent?** For the given values, the minimum points are (4, 6) or (6, 4). These represent the rectangles with the least perimeter if only integer values are allowed. Actually the rectangle with the least perimeter is a square with sides $\sqrt{24}$ and perimeter $4\sqrt{24} \approx 19.6$.

(Graph #3) **When the points are connected and the curve is extended, do you think the curve will ever cross the y-axis?** Yes (and no). **What do you think this means?** The line ($y = 24$) will cross the y-axis at (0, 24), which means that the area is always 24. But in the context of the problem, if the length is zero, the rectangle would have no area. This can be noted with an open circle at the point (0, 24).

Practice

Pairs/ Individuals

- SP8*
Graphing
Rectangles

- Students draw rectangles with areas of 18 sq. units on a coordinate grid. They label the coordinates of the vertices and lengths of the sides, and find the perimeter.

How are the coordinates related to the lengths of the sides? The lengths of the vertical segments are the differences in the y-coordinates, and the lengths of the horizontal segments are the differences in the x-coordinates.

CLOSURE

Whole Class

- SP1*
Ready, Set

- Review the goals and standards for the lesson.

13.1 Rectangles: Fixed Area

SELECTED SOLUTIONS				
SP1 Warmup	1. Point Q: (-5, 3) 2. Point R: (3, 3)	3. Point S: (3, -2) 4. Point T: (-5, -2)	5. Perimeter = 26 units 6. Area = 40 square units	
SP3 Area = 12 units ²	Length (linear units)	Width (linear units)	Perimeter (linear units)	Area (square units)
	1	12	26	12
	2	6	16	12
	3	4	14	12
	4	3	14	12
	6	2	16	12
	12	1	26	12
SP4 Fixed Area Table	Length	Width	Perimeter (linear units)	Area (sq. units)
	1	24	50	24
	2	12	28	24
	3	8	22	24
	4	6	20	24
	6	4	20	24
	8	3	22	24
	12	2	28	24
24	1	50	24	
SP5-7 Fixed Area Graphs 1-3	<ol style="list-style-type: none"> No, it will not cross the y-axis. The product of length and width is 24, so as the length gets close to zero, the width gets infinitely long. For the given values, the minimum points are (4, 6) or (6, 4). These represent the rectangles with the least perimeter if only integer values are allowed. Actually the rectangle with the least perimeter is a square with sides $\sqrt{24}$ and perimeter $4\sqrt{24} \approx 19.6$ Yes (and no). The line ($y = 24$) will cross the y-axis at (0, 24), which means that the area is always 24. But in the context of the problem, if the length is zero, the rectangle would have no area. This can be noted with an open circle at the point (0, 24). 			
SP8 Graphing Rectangles	<ol style="list-style-type: none"> 1-4. Answers will vary. 5. The lengths of the vertical segments are the differences in the y-coordinates, and the lengths of the horizontal segments are the differences in the x-coordinates. 			

STUDENT PAGES

RECTANGLES: FIXED AREA

Ready (Summary)

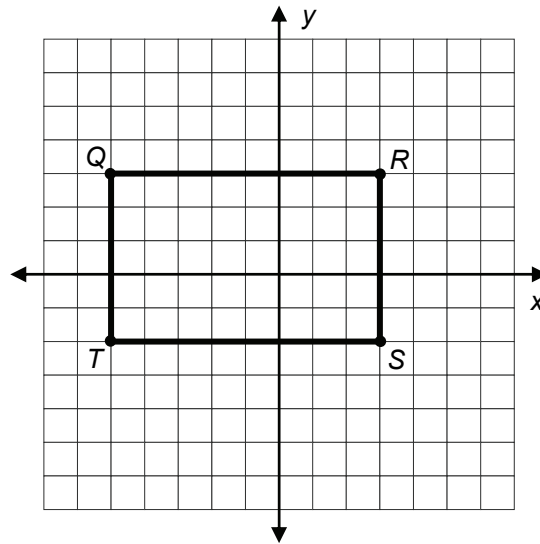
We will review how to find perimeters and areas of rectangles. We will create different rectangles with a fixed area, record the data in a table, and then graph the data.

Set (Goals)

- Find lengths of horizontal and vertical line segments on a coordinate system.
- Identify and graph coordinates in four quadrants of the coordinate plane.
- Find the perimeters of rectangles with fixed areas.
- Use formulas for the perimeter and area of rectangles to solve problems.
- Interpret the meaning of graphs.

Go (Warmup)

Find the coordinates of points Q , R , S , and T . Then, find the perimeter and area of the rectangle.

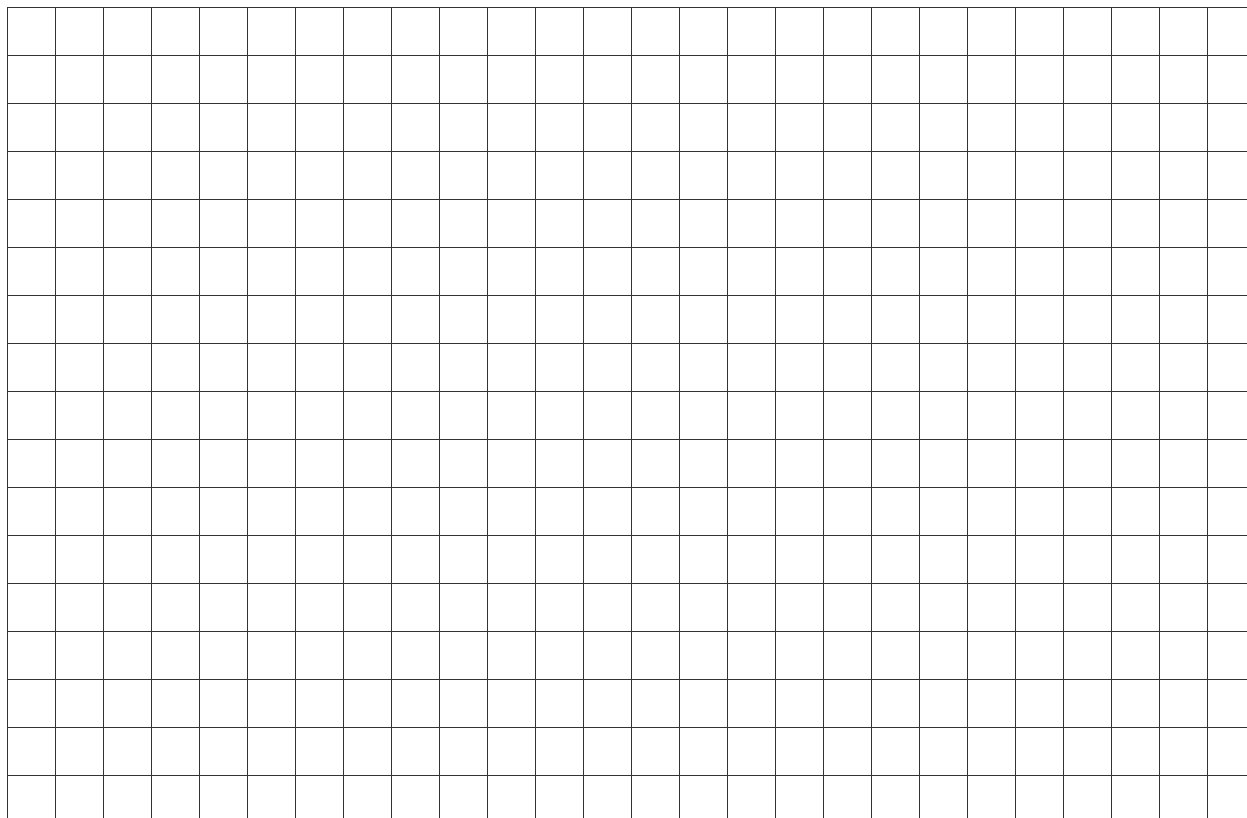


1. Point Q : (____, ____)
2. Point R : (____, ____)
3. Point S : (____, ____)
4. Point T : (____, ____)
5. Perimeter of rectangle $QRST$: _____
6. Area of rectangle $QRST$: _____

13.1 Rectangles: Fixed Area

$$\text{AREA} = 12 \text{ units}^2$$

Sketch rectangles on this grid paper with an area of 12 square units.

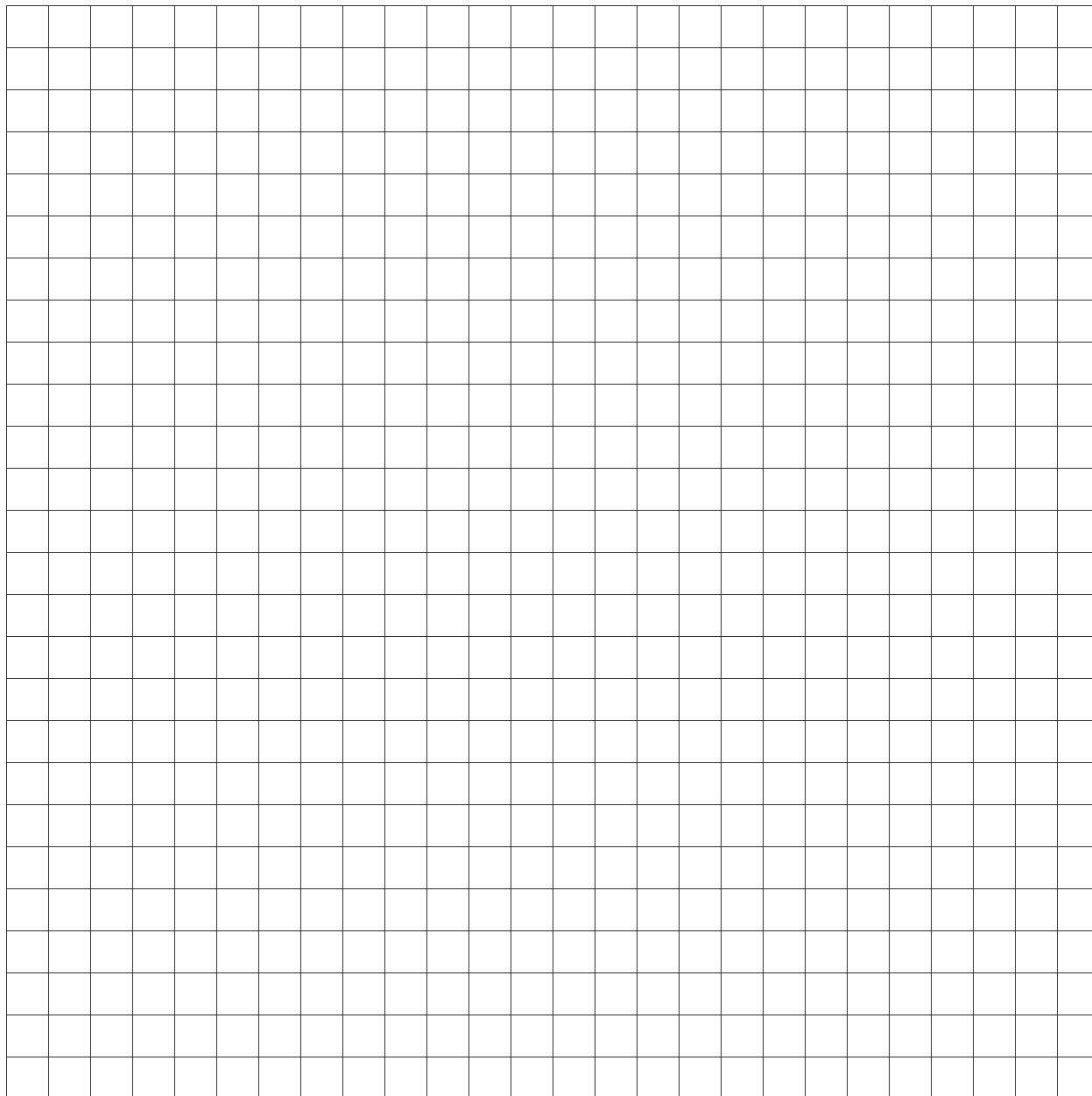


Record the dimensions of the rectangles you drew with the lengths in ascending order.

Length (linear units)	Width (linear units)	Perimeter (linear units)	Area (square units)

FIXED AREA RECTANGLES

Use square tiles to create all the rectangles that have an area of 24 square units and sketch them on this grid paper.



How many rectangles did you find? _____

How are the dimensions of the rectangles related to their areas?

13.1 Rectangles: Fixed Area

FIXED AREA TABLES

Use the tables to record the information about the rectangles you found with an area of 24 square units. Let the horizontal distance be the length and let the vertical distance be the width.

Table 1	
Length	Width

Table 2	
Length	Perimeter

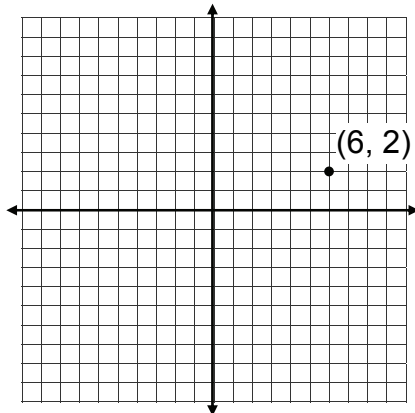
Table 3	
Length	Area

What conjectures can you make about the perimeters of rectangles with fixed areas?

GRAPHING RECTANGLES

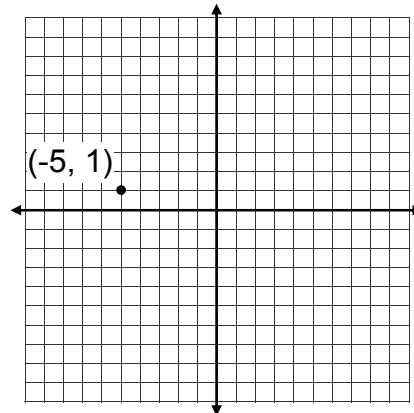
- Draw four different rectangles whose area is 18 square units. One of the vertices for each rectangle is given.
- Label the coordinates of the vertices.
- Find the perimeter for each rectangle.

1.



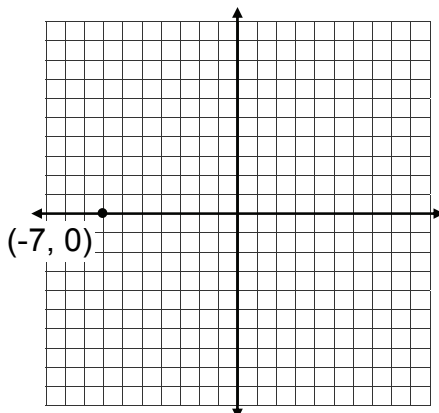
Perimeter = _____

2.



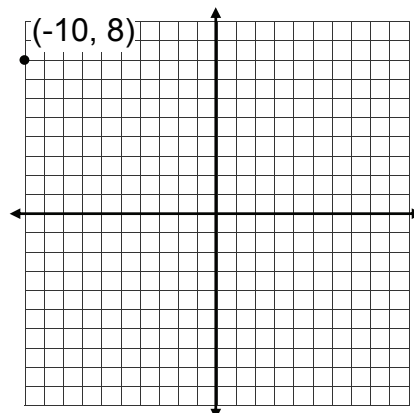
Perimeter = _____

3.



Perimeter = _____

4.



Perimeter = _____

5. Explain how to find the horizontal length in problem 1 using the coordinates of the vertices. _____
6. Explain how to find the vertical length in problem #2 using the coordinates of the vertices. _____